Pension Liability Valuation and Asset Allocation in the Presence of Funding Risk

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Defined benefit pension liabilities are usually computed by discounting promised future pension payments using the yields on either risk-free or AA-rated bonds. We argue that a pension plan in financial distress should use discount rates that reflect the inherent funding risk. We propose a new valuation approach that utilizes the term structure of funding-risk-adjusted discount rates. These discount rates depend on the current asset allocation of the pension plan which affects expected future funding ratios. We show that an optimal asset allocation which accounts for this dependency varies in a highly nonlinear way with the initial funding ratio of the pension plan. In particular, the optimal allocation to stocks is higher than conventionally determined when the level of underfunding is severe, but lower when the level of underfunding is only moderate.

Key words: term structure, funding risk, funding spread, asset-liability management, pension plan

1. Introduction

One of the fundamental insights of modern financial economics states that promised future cash flows should be discounted using (a term structure of) discount rates that appropriately reflect the risks underlying those cash flows. This paper proposes a term structure of discount rates for valuing pension benefits in the presence of funding risk. This is the risk that the future funding positions of a defined benefit (DB) pension plan are insufficient to guarantee the promised pension benefits.[1] It resembles the well-known credit risk which describes the risk that the issuer of a corporate bond fails to pay the

[1] Although corporate DB pension plans are increasingly being replaced by defined contribution plans, DB plan assets are still substantial and amounted to $6.4 trillion in the U.S. alone at the end of 2006 (Watson Wyatt, 2007).

interest or principal. It is now standard practice to value corporate bonds and credit derivatives using a term structure of issuer-specific credit spreads over the term structure of interest rates (see, for example, Das and Sundaram, 2000). By contrast, DB pension liabilities are valued using discount rates which completely disregard funding risk, as previously recognized by Petersen (1996) and Ippolito (2002). Funding risk is plan specific (in the same way that credit risk is issuer specific), since it depends on the current funding ratio of plan assets to plan liabilities, on the sponsor's ability to close funding gaps over time, and on decisions of the sponsor that affect future funding ratios.

In this paper, we will derive a term structure of funding spreads which appropriately reflects the funding risk of the pension plan. The funding spreads are endogenous (as recognized by Duffie and Singleton, 1999) in the sense that they are dependent on expected future funding ratios and, thus, on the current asset allocation of the pension plan. Petersen (1996) predicted our result: "As the firm shifts the pension assets from low risk assets (cash) to higher risk assets (stocks), the discount rate will rise only if the pension liability does not remain risk free." When the optimal asset allocation depends on an objective function in the funding-risk-adjusted (FRA) liability, required discount rates and optimal portfolio weights are interdependent and can be jointly determined in a single optimization step. We illustrate this by introducing the term structure of funding spreads in the asset-liability model proposed by Hoevenaars et al. (2008), which, in turn, builds on the asset allocation models of Campbell et al. (2003) and Campbell and Viceira (2005). Hoevenaars et al. consider an objective function in the conditionally expected utility of the terminal funding ratio. We modify that objective function by introducing FRA liabilities and show that optimizing this new function with respect to the asset allocation will automatically generate the desired term structure of funding spreads and hence a value for the pension liability that appropriately reflects funding risk. However, in contrast with the optimal asset allocation obtained by Hoevenaars et al. (2008), the optimal portfolio implied by our methodology will depend on the initial funding ratio of the pension plan. This should come as no surprise, since the initial funding ratio affects the term structure of funding spreads which is determined jointly with the optimal portfolio.

2. Background

2.1 Pension Accounting

Before discussing our model in more detail, it is worth taking a closer look at existing practice in pension liability valuation. We need to distinguish between practice before and after the release by the U.S. Financial Accounting Standards Board (FASB) of Statement No. 87 concerning "Employers' Accounting for Pensions" which came into force for the fiscal years after December 15, 1986. Prior to FAS 87, companies sponsoring a DB pension plan used a wide array of assumptions to determine the market value of plan liabilities. Feldstein and Mørck (1983) showed that the discount rates assumed for the valuation of pension liabilities ranged from 5 percent to 10.5 percent for a sample of large manufacturing firms in 1979. The authors discover evidence that the precise choice was determined by a sponsoring company's trade-off between the tax advantage of a low discount rate and the cosmetic benefit to the annual report arising from a high discount rate; high discount rates could also be used to escape a Department of Labor request for additional contributions to the pension plan. Bodie et al. (1987) found evidence that more profitable firms use lower discount rates to calculate pension liabilities in an attempt to smooth corporate earnings. Overall, before FAS 87, the choice of discount rate appeared to be guided more by strategic management considerations than by the exercise of fiduciary responsibility towards plan beneficiaries by plan sponsors.

FAS 87 reduced the discretion sponsoring companies had over the choice of the discount rate. The Statement requires that "assumed discount rates shall reflect the rates at which the pension benefits could be effectively settled." The discount rates regularly published by the Pension Benefit Guaranty Corporation (PBGC) and used to value the liabilities of terminated pension plans, satisfied this condition. However, FAS 87 also allows the company "to look to rates of return of high-quality fixed income investments currently available and expected to be available during the period to maturity of the pension benefits." In practice, sponsoring companies often use the average yield to maturity on long-term corporate bonds with a Moody's AA rating (Coronado and Sharpe, 2003).[2] However, this practice still leaves some degree of discretion and this can be exercised strategically to manipulate earnings. Bergstresser et al. (2006) offer a post-FAS 87 analysis of pension assumptions and show that the

[2] This choice is also consistent with U.K. and international accounting standards FRS 17 and IAS 19, respectively.

expected return on plan assets, another assumption required by FAS 87, tends to be used to manipulate reported earnings. Similarly, Cocco and Volpin (2007) show that insider trustees in the U.K, who are also executive directors of the sponsoring company, tend to act in favor of the shareholders of the sponsor rather than in the interests of the pension plan members.

We propose a method of liability valuation which will generate heterogeneous, pension-plan-specific, discount rates as in the pre-FAS 87 period, but in a systematic and standardized way as in the post-FAS 87 period. Our method avoids any discretionary freedom with respect to the choice of the discount rate and thus effectively prevents manipulations of the form detected by Bergstresser et al. (2006) and Cocco and Volpin (2007).

2.2 Why Consider Funding Risk?

Feldstein and Seligman (1981) argue that promised future pension benefits are a substitute for current wages. Consequently, if the promise is not fully funded, then a company introducing a pension plan in exchange for lower current wages will create accounting profits. If the stock market correctly values the underfunded pension obligations, the company's share price will drop by the extent of underfunding. In this case, shareholders will not be fooled by temporary accounting profits and, hence, will leave their lifetime consumption plan unchanged. However, if the market incorrectly values underfunded pension liabilities, shareholders might interpret temporary accounting profits as an increase in permanent income and increase consumption accordingly. Contrary to the findings of studies of the pre-FAS 87 period by Feldstein and Seligman (1981) and Bulow et al. (1987), recent studies of the post-FAS 87 period by Coronado and Sharpe (2003) and Franzoni and Marín (2006) find that the market does not correctly value firms with a DB pension plan. Coronado and Sharpe find evidence that all companies with a DB plan are overvalued, while Franzoni and Marín show that the market only overvalues companies with underfunded DB pension liabilities. Thus, the Feldstein and Seligman argument applies and shareholders are at risk of suboptimally cutting savings.

A regulator adopting our proposed method of pension liability valuation would help to increase shareholder awareness of possible future funding gaps, since the funding spreads that emerge from our method reflect both future underfunding probabilities and expected funding ratios conditional on underfunding. Reporting the term structure of funding spreads would increase transparency and help shareholders make optimal consumption decisions.

Pension plan members are a second group of stakeholders with a clear interest in the FRA value of their pension promise, since the future pension payments they expect to receive will also influence their lifetime savings and consumption choices. If the term structure of funding spreads indicates considerable funding risk, then the member might decide to compensate through increased private pension savings at the cost of reduced consumption. Again, transparency would help plan beneficiaries make optimal choices.

Transparency with respect to future underfunding probabilities will benefit the third key stakeholder of the pension plan, namely the sponsoring company itself. Rauh (2006) shows that, for companies facing financial constraints, capital expenditures decline by the amount of mandatory contributions to their DB pension plans. The term structure of funding spreads immediately highlights possible future financial constraints arising from the current decisions of the plan sponsor in respect of the funding level, the magnitude of contributions, the generosity of benefits and the asset allocation. Henceforth, the plan sponsor will not be surprised by the need to make future contributions to the plan, nor by the consequential requirement to curtail corporate investment.

3. Valuation in the Presence of Funding Risk

3.1 Funding Spreads

We are interested in obtaining the current time-t value, P_t^s, of a pension benefit payment, B, at some future time $t+s$. B is stochastic because it depends on the ability of the plan sponsor to cover its liabilities at time $t+s$. The expected pension benefit payment might therefore be lower than the promised benefit. We assume that underfunding occurs with probability π_t^s. In the case of underfunding, only a recovery fraction, λ_t^s, of the promised pension benefit, B, will be paid off. Under these assumptions, we can apply the fundamental equation of asset pricing (see Cochrane, 2001) to obtain the current value of the future pension payment as

$$P_t^s = E_t\left[M_{t+s}\text{Payoff}_{t+s}\right] = E_t\left[M_{t+s}\right]E_t\left[\text{Payoff}_{t+s}\right] + \text{cov}_t\left(M_{t+s}, \text{Payoff}_{t+s}\right) \quad (1)$$

where $$E_t\left[M_{t+s}\right] = \left(1 + Y_t^s\right)^{-s} \tag{2}$$

$$E_t\left[\text{Payoff}_{t+s}\right] = \left(1 - \pi_t^s\right)B + \pi_t^s \lambda_t^s B\,. \tag{3}$$

M_{t+s} denotes the s-period stochastic discount factor (SDF), which equals the inverse of the risk-free rate for a maturity of s periods in conditional expectation as shown in (2). We use conditional expectations to allow for time variation in investment opportunities. The expected pension payoff is derived in (3) as the probability-weighted sum of the pension payoffs in the states of over- and underfunding.

The conditional covariance term in (1) depicts the risk correction (Cochrane, 2001) term. For our purposes, we find it more convenient to replace this additive term with a multiplicative term, $\left(1 + \theta_t^s\right)^{-s}$, where θ_t^s defines the funding-risk premium (FRP). Then we can rewrite (1) as

$$P_t^s = \frac{\left(1 - \pi_t^s\right)B + \pi_t^s \lambda_t^s B}{\left(1 + Y_t^s\right)^s \left(1 + \theta_t^s\right)^s} = \frac{B}{\left(1 + Y_t^s\right)^s \left(1 + \Delta_t^s\right)^s} \tag{4}$$

where $$\left(1 + \theta_t^s\right)^{-s} = 1 + \text{cov}_t\left(M_{t+s}, \text{Payoff}_{t+s}\right) / \left(E_t\left[M_{t+s}\right] E_t\left[\text{Payoff}_{t+s}\right]\right) \tag{5}$$

$$\left(1 + \Delta_t^s\right)^{-s} = \left(1 + \theta_t^s\right)^{-s}\left(1 - \pi_t^s + \pi_t^s \lambda_t^s\right). \tag{6}$$

The first equality in (4) follows from replacing (2), (3) and (5) in equation (1). For the second equality in (4), we use the promised pension benefit in the numerator which then needs to be discounted by a FRA discount factor using the funding spread, Δ_t^s, defined in (6), over the yield on a risk-free bond with maturity s.[3] For varying s, (6) defines a term structure of funding spreads. The funding spread increases with a higher FRP. For a given FRP, the funding spread increases with an increasing conditional underfunding probability and decreases with an increasing recovery fraction in the case of underfunding.

The term structure of funding spreads is completely described by π_t^s, λ_t^s and θ_t^s. We define

$$\pi_t^s = \Pr_t\left(\frac{A_{t+s}}{L_{t+s}^0} < \tau\right) \tag{7}$$

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[3] The reader will notice that our approach of deriving funding spreads resembles the derivation of credit spreads in credit risk models. Das and Sundaram (2000) provide a discrete time reduced-form model which leads to credit spreads of the form (6).

$$\lambda_t^s = \frac{1}{\tau} E_t\left[\frac{A_{t+s}}{L_{t+s}^0} \mid \frac{A_{t+s}}{L_{t+s}^0} < \tau\right]. \tag{8}$$

According to (7), underfunding at horizon s occurs when the funding ratio of assets, A_{t+s}, over liabilities, L_{t+s}^0, falls below a funding threshold, τ. L_{t+s}^0 is the present value of all future pension payments promised by the plan sponsor, discounted using zero funding spreads (which explains the superscript). While (7) defines the conditional underfunding probability, (8) defines the recovery fraction as the expected funding ratio conditional on underfunding. Because the recovery fraction is scaled by a funding threshold, τ, it is possible to have threshold levels larger than unity (without violating the inequality $\lambda_t^s \leq 1$) in the case where a regulatory authority requires the sponsoring company to maintain a funding buffer.[4] It is important to recognize that π_t^s and λ_t^s depend on the future funding ratio which itself depends on the current asset allocation chosen by the pension plan. Thus, *the term structure of funding spreads is a function of the chosen asset allocation*.

If the pension plan is independent of the sponsoring company and the latter is under no obligation to close any funding gap, then $\tau = 1$ would seem to be a sensible choice. If the regulatory authority obliges the sponsoring company to close any funding gap, the funding threshold could be modeled as $\tau = 1 - N_{t+s}/L_{t+s}^0$ such that $\pi_t^s = \Pr_t\left((A_{t+s} + N_{t+s})/L_{t+s}^0 < 1\right)$ where N_{t+s} denotes the future net worth of the sponsoring company.[5] In this case, underfunding only occurs whenever the sum of the pension plan assets and the net worth of the sponsoring company falls below the value of the pension liability.

3.2 Funding-Risk Premia

The risk premium for being exposed to funding risk at horizon s is determined by (5). We can simplify this expression if we are willing to reduce the state space to the two possible states of the world

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[4] In the Netherlands, for example, the pension regulator requires pension plans to be 105% funded at all times (τ = 1.05).

[5] Similarly, N_{t+s} could be set at the level of liabilities that are covered by a pension guarantee fund such as the Pension Benefit Guaranty Corporation (PBGC) in the U.S. or the Pension Protection Fund (PPF) in the U.K. in case of default by the plan sponsor. However, because of large concentration risks and moral hazard affecting the behavior of the covered companies, the guarantee fund is itself subject to underfunding and default risk unless the government underwrites any funding gap (see McCarthy and Neuberger, 2005).

which are relevant for the determination of the pension benefit payoff, namely the states of over- and underfunding.[6] In this case, $E_t[M_{t+s}] = (1-\pi_t^s)M_{t+s}^o + \pi_t^s M_{t+s}^u$. Then (1) can be rewritten as $P_t^s =$ $E_t[M_{t+s}\text{Payoff}_{t+s}] = (1-\pi_t^s)M_{t+s}^o B + \pi_t^s M_{t+s}^u \lambda_t^s B$. From equating this expression with the first expression in (4) we obtain

$$\left(1+\theta_t^s\right)^{-s} = \frac{\left(1-\pi_t^s\right)}{\left[\left(1-\pi_t^s\right)+\pi_t^s\lambda_t^s\right]}\frac{M_{t+s}^o}{E_t[M_{t+s}]} + \frac{\pi_t^s\lambda_t^s}{\left[\left(1-\pi_t^s\right)+\pi_t^s\lambda_t^s\right]}\frac{M_{t+s}^u}{E_t[M_{t+s}]}. \tag{9}$$

The FRP is now completely determined by π_t^s, λ_t^s and the ratios of the SDFs (in the states of over- and underfunding) to the expected SDF. For a given stochastic discount factor, these ratios are much easier to determine than the conditional covariance term in the original FRP equation (5).

3.3 Stochastic Discount Factor

The stochastic discount factor only affects the FRP (9) which is a part of the funding spread (6). We assume that the SDF is the result of negotiation between the stakeholders of the defined benefit scheme. Suppose, we assume the stakeholders agreed to use a consumption-based asset model (see Cochrane, 2001) which implies SDFs in the states of over- ("o") and underfunding ("u") of the form

$$M_{t+s}^o = \beta^s\left(\frac{C_{t+s}^o}{C_t}\right)^{-\gamma} = \beta^s\left(g_{t+s}^o\right)^{-\gamma} \quad \text{and} \quad M_{t+s}^u = \beta^s\left(\frac{C_{t+s}^u}{C_t}\right)^{-\gamma} = \beta^s\left(g_{t+s}^u\right)^{-\gamma} \tag{10}$$

where $g_{t+s}^o = C_{t+s}^o / C_t$ and $g_{t+s}^u = C_{t+s}^u / C_t$ denote consumption growth in the states of over- and underfunding (corresponding to states of boom and slump, respectively), β denotes the subjective time-discount factor and γ the coefficient of relative risk aversion for the representative pension plan member. We know from (9) that the FRP is positive (i.e., $\theta_t^s > 0$), if $M_{t+s}^u > M_{t+s}^o$ which is equivalent to $g_{t+s}^o > g_{t+s}^u$. This is likely to be the case when overfunding corresponds to a state of high asset values and the representative pension plan member invests in the same asset classes as the pension

[6] A less restrictive state space would not generate any additional insights, but would complicate the computation of the funding-risk premium. In the credit risk literature, the risk premium is often specified in an ad hoc manner. In comparison, our specification of the risk premium is much better motivated and leads to an expression which can be readily applied in practice.

plan and increases his consumption when his wealth is high. Given (10), we can determine the remaining unknown elements of the FRP, (9), as

$$\frac{M^o_{t+s}}{E_t[M_{t+s}]} = \frac{\left(g^o_{t+s}\right)^{-\gamma}}{\left(1-\pi^s_t\right)\left(g^o_{t+s}\right)^{-\gamma} + \pi^s_t\left(g^u_{t+s}\right)^{-\gamma}} = \left[\left(1-\pi^s_t\right) + \pi^s_t\left(\frac{g^o_{t+s}}{g^u_{t+s}}\right)^{\gamma}\right]^{-1} \quad (11)$$

$$\frac{M^u_{t+s}}{E_t[M_{t+s}]} = \frac{\left(g^u_{t+s}\right)^{-\gamma}}{\left(1-\pi^s_t\right)\left(g^o_{t+s}\right)^{-\gamma} + \pi^s_t\left(g^u_{t+s}\right)^{-\gamma}} = \left[\left(1-\pi^s_t\right)\left(\frac{g^o_{t+s}}{g^u_{t+s}}\right)^{-\gamma} + \pi^s_t\right]^{-1}. \quad (12)$$

[7]

For $g^o_{t+s} > g^u_{t+s}$, (11) will be smaller than unity, while (12) will be larger than unity. The FRP, θ^s_t, is known, once we know $\phi = g^o_{t+s} / g^u_{t+s}$, the ratio of consumption growths in the states of over- and underfunding, respectively. We will assume this ratio is constant, with $\phi > 1$. In the empirical section of the paper, ϕ will be varied in order to assess the responsiveness of the FRPs.

It is useful to derive some comparative statics results from (9) for θ^s_t. First, we can see that θ^s_t is zero, whenever $\pi^s_t = 0$ or $\pi^s_t = 1$. Thus, if one of the two possible states of the world occurs with certainty, the FRP is zero, whether or not this state is favorable or unfavorable for the pension plan member. For conditional underfunding probabilities between the two extreme outcomes, $0 < \pi^s_t < 1$, we can show (after some simple but tedious calculations) that the FRP increases with an increasing underfunding probability (for fixed s and λ^s_t) when $\phi^{-\gamma} > \lambda^s_t \left(\pi^s_t\right)^2 \left(1-\pi^s_t\right)^{-2}$. The FRP decreases with an increasing underfunding probability when the inequality is reversed. If $\phi^{-\gamma} = \lambda^s_t \left(\pi^s_t\right)^2 \left(1-\pi^s_t\right)^{-2}$, this corresponds with a point of maximum uncertainty about the future state of the world, so pension plan members will demand the highest risk premium here. The FRP decreases with increasing maturity, s, and recovery fraction, λ^s_t, and becomes zero for $\lambda^s_t = 1$. All comparative statics results conform with a priori expectations.

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[7] We would obtain very similar expressions for alternative stochastic discount factors like the Campbell and Cochrane (1999) SDF, which have been shown to work better in empirical work than the SDF of the consumption based model.

4. Illustration

4.1 Assets and Liabilities

We consider a stylized DB pension plan which is completely independent of the sponsoring company. The latter is under no obligation to close any funding gap. Correspondingly, the funding threshold is chosen as $\tau = 1$. When valuing pension liabilities of the stylized pension plan, we make a number of simplifying assumptions. First, we assume that all future promised pension benefit payments, B, are constant in nominal terms in order to abstract from inflation risk.[8] Second, like Hoevenaars et al. (2008), we assume that the plan is sufficiently large that longevity risk is diversified away. Third, also in line with these authors, we assume that the maturity of the pension liability is constant (which holds for a pension plan in a stationary state where the distribution of age cohorts and accrued benefit rights of plan members remains constant over time). Fourth, again like Hoevenaars et al., we assume that new contributions to the plan exactly offset any increase in accrued pension rights. The overarching purpose of these assumptions is to allow us to focus on the change in liability value arising exclusively from changes in the yield curve. Then, the time $t+s$ value of the pension liability follows as

$$L_{t+s} = \sum_{m=1}^{M} P_{t+s}^{m} . \tag{13}$$

where M is the liability horizon which defines the maturity of the pension plan and P_{t+s}^{m} is determined from (4). From (13) the gross liability return is obtained as

$$R_{t+s}^{L} = \frac{L_{t+s}}{L_t} = \frac{\sum_{m=1}^{M} P_{t+s}^{m}}{\sum_{m=1}^{M} P_{t}^{m}} = \sum_{m=1}^{M} \frac{P_t^m}{\sum_{n=1}^{M} P_t^n} R_{t+s}^{m} = \sum_{m=1}^{M} v_t^m R_{t+s}^{m} = v_t' R_{t+s} \qquad \text{with} \quad v_t^m = \frac{P_t^m}{\sum_{n=1}^{M} P_t^n} . \tag{14}$$

The return on liabilities is equal to the return on a value-weighted portfolio of zero-coupon bonds. For subsequent purposes, we denote the vector of variables $\left(L_{t+s}, R_{t+s}^{L}, R_{t+s}\right)$ with a zero superscript, $\left(L_{t+s}^{0}, R_{t+s}^{L,0}, R_{t+s}^{0}\right)$, when used in connection with zero funding spreads at all maturities. In this case, promised and expected pension benefits are always equal. In the presence of funding risk, we will use

[8] This is a fairly realistic assumption. In the U.S., for example, companies are obliged to publish pension liabilities which are calculated on a nominal basis. Inflation risk has been considered elsewhere (for example, in Hoevenaars et al., 2008).

the superscript Δ in order to indicate positive funding spreads, $\left(L^{\Delta}_{t+s}, R^{L,\Delta}_{t+s}, R^{\Delta}_{t+s}\right)$. We denote L^{0}_{t+s} the risk-free (RF) liability – which is the value of the promised pension found by discounting promised future pension payments using the term structure of interest rates – and L^{Δ}_{t+s} the FRA liability – obtained by discounting the same promised payments using the term structure of FRA discount rates.

Let A_{t+s} denote the value of the pension plan assets at time $t+s$. Then $A_{t+s} = A_t R^{A}_{t+s}$, where $R^{A}_{t+s} = R^{f}_{t+s} + w'_t R^{e}_{t+s}$, is the return of the portfolio which includes a risk-free return, R^{f}_{t+s}. R^{e}_{t+s} denotes a vector of returns of risky assets in excess of R^{f}_{t+s} and w_t a vector of corresponding portfolio weights. Having defined the assets and liabilities of the pension plan, the funding ratios follow immediately as $F^{0}_{t+s} = A_{t+s} / L^{0}_{t+s}$ and $F^{\Delta}_{t+s} = A_{t+s} / L^{\Delta}_{t+s}$.

4.2 Asset Allocation

In line with both Van Binsbergen and Brandt (2007) and Hoevenaars et al. (2008), we assume that the pension plan maximizes the conditional time-t expectation of a utility function in the terminal funding ratio at horizon k. This is a natural extension of an objective function in the expected utility of final wealth – considered, for example, by Campbell and Viceira (2005) – to the case of an institutional investor who cares about liabilities. Assuming power utility with a constant coefficient of relative risk aversion, γ, and a normal distribution for cumulative log funding ratio returns, s_{t+k}, the optimization problem is equivalent to the following mean-variance optimization problem (see Campbell and Viceira, 2002)

$$w_t = \arg\max_{w_t} \left\{ E_t\left[s_{t+k}\right] + \frac{1}{2}(1-\gamma)V_t\left[s_{t+k}\right] \right\}. \quad (15)$$

We define the cumulative log funding ratio return as $s^{0}_{t+k} = \ln\left(R^{A}_{t+k} / R^{L,0}_{t+k}\right)$ if based on RF liabilities and as $s^{\Delta}_{t+k} = \ln\left(R^{A}_{t+k} / R^{L,\Delta}_{t+k}\right)$ if based on FRA liabilities.[9] We denote the resulting asset allocations as

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[9] These log returns are derived in appendix A. The appendix also derives the conditional probability of underfunding (7) and the recovery fraction (8) in terms of log returns. It turns out that both expressions only depend on the ratio of the initial funding ratio, F^0, to the funding threshold, τ, not the absolute value of the two parameters.

w_t^0 and w_t^Δ. The former allocation is due to Hoevenaars et al. (2008), while the latter allocation is our adaptation to the case where funding risk is present.[10] Although the two asset allocation problems appear similar at first sight, it should by now be clear that solving for w_t^Δ is much more demanding than solving for w_t^0 because of the presence of endogenous funding spreads which depend on the chosen asset allocation. Therefore w_t^Δ has to be obtained by numerical methods while w_t^0 is available in closed from (Hoevenaars et al., 2008). w_t^0 is a weighted average of two components, one related to speculative asset demand and the other to liability-hedging demand. We will use w_t^0 as a benchmark for comparing the results obtained from w_t^Δ in the empirical part of the paper.

4.3 Return Dynamics

We follow Campbell et al. (2003), Campbell and Viceira (2005), Van Binsbergen and Brandt (2007) and Hoevenaars et al. (2008) and model the return dynamics by means of a vector autoregression with one lag, VAR(1). From equations (14) and (15) it is clear that we need k-period ahead forecasts of log asset returns and up to $(k+m)$-period ahead forecasts of the log returns on a default-free zero-coupon bond with maturity m, for $m = 1,\ldots,M$. Since M will be small in our application (we use the CRSP Fama-Bliss zero-coupon data where the maximum maturity equals five years), we will not specify an arbitrage-free term structure model that generates yields with long maturities. At their simplest, the liabilities of a pension plan could consist of a single discounted pension payment at maturity m. Adding payments with additional maturities to the liability structure of the pension plan will make little difference to the resulting asset allocation unless asset and liability log return correlations vary substantially with maturity. For this reason we can omit a model for the term structure of interest rates from the current analysis.

[10] Since the conditional underfunding probability, π^s_t, and the recovery fraction, λ^s_t, in the case of underfunding are closely related to the risk measures Value at Risk (VaR) and Expected Shortfall (ES), we can justify Campbell and Viceira’s (2005) assumption of a buy-and-hold investment strategy for our approach: both VaR and ES are usually computed under the assumption that the chosen asset allocation will be maintained over the forecast period, as emphasized by Cuoco et al. (2005).

Like Campbell and Viceira (2005), we assume that the risk premium on quarterly stock returns is driven by the dividend-price ratio, q_t. But unlike these authors, we do not include the spread between the yield on a 5-year zero-coupon bond and the nominal short-term interest rate. This is because we are already using log returns on zero-coupon bonds with maturities up to M years in our regression for the liability return vector, r^0_{t+1}. We do, however, use the log return on T-bills, r^f_{t+1}, and stock and bond excess log returns, r^e_{t+1}. The next subsection discusses the data in more detail.

Collecting all required data in the vector $z'_{t+1} = \left[r^f_{t+1}, r'^e_{t+1}, q_{t+1}, r'^0_{t+1}\right]$ for quarters $t = 1,\ldots,Q-1$, we specify a homoskedastic VAR(1) model $z_{t+1} = \Phi_0 + \Phi_1 z_t + \varepsilon_{t+1}$ with $\varepsilon_{t+1} \sim N(0,\Sigma)$. Stability (and thus stationarity) of the VAR(1) system requires that the maximum eigenvalue of Φ_1 is smaller than unity. Campbell and Viceira (2004) derive the conditional first and second moments of cumulative k-period log returns (and state variables) implied by the VAR(1) model which are

$$E_t\left[z_{t+1} + \ldots + z_{t+k}\right] = \left[\sum_{i=0}^{k-1}(k-i)\Phi_1^i\right]\Phi_0 + \left[\sum_{i=1}^{k}\Phi_1^i\right]z_t \qquad (16)$$

$$V_t\left[z_{t+1} + \ldots + z_{t+k}\right] = \sum_{j=1}^{k}\left(\left(\sum_{i=0}^{j-1}\Phi_1^i\right)\Sigma\left(\sum_{i=0}^{j-1}\Phi_1^i\right)'\right). \qquad (17)$$

Note that these conditional moments are investment horizon (k) dependent. In order to generate representative asset allocations for our sample, we will evaluate (16) at the average value of the state variables in Section 5. Using (16) and (17) we can generate all required moments.

5. Empirical Analysis

5.1 Data and Estimated Return Dynamics

We consider a simple pension plan which faces pension payments in 1, 2, 3, 4 and 5 years' time; thus, $M = 5$ years in our model. The choice of M is dictated by the availability of zero-coupon bond price data which we obtained from the CRSP Fama and Bliss files for the period 1952:Q2 –2005:Q4.[11] We use the same time period for our other quarterly data.

In line with Campbell and Viceira (2005), we assume the pension plan only considers investing in the principal asset classes of cash, bonds and stocks. We use the same quarterly data on interest rates, bond and stock returns as Goyal and Welch (2007). Thus, we compute continuously compounded quarterly returns of the S&P500 index including dividends. The dividend-price ratio is based on 12-month moving sums of dividends paid on the S&P500 index. Bond returns refer to long-term government bonds. Stock and bond excess returns are with respect to nominal T-bill returns. We use the nominal T-bill rate because the pension plan promises pension payments is assumed to be in nominal terms.

Mean yields of zero-coupon bonds increase from 5.56% for a one-year maturity to 6.14% for a five-year. maturity. Short-term yields are slightly more volatile than long-term yields. Based on these yields, we compute prices $P_t^m = \left(1 + Y_t^m\right)^{-m}$ for every period t and maturity m and constant-maturity returns $R_{t+k}^{0,m} = P_{t+k}^m \big/ P_t^m$. The mean of constant-maturity zero-coupon returns is approximately zero at all maturities. The volatility increases from 1.75% for the shortest maturity to 6.24% for the longest maturity. Due to the constant-maturity assumption, the value of the pension plan liability hardly changes in expectation over time, but will, given these bond return volatilities, be subject to considerable interest rate risk.[12]

The VAR(1) model for the asset and liability return dynamics is estimated by OLS.[13] The maximum eigenvalue of the matrix Φ_1 is 0.9805. Thus, the system is both stable and stationary. The estimates are in line with earlier results obtained by Campbell at al. (2003), Campbell and Viceira (2005) and Hoevenaars et al. (2008). We confirm previous findings of these authors regarding the impact of the log dividend-price ratio on stock excess log returns. High dividend-price ratios are negatively cor-

[11] Note also, that we can still generate term structures of funding spreads from (15) for horizons beyond M = 5.

[12] Table B1 in appendix B shows annualized descriptive statistics for the data.

[13] Table B2 in appendix B shows the estimation results for the VAR(1) parameter matrix [Φ_0, Φ_1]. Table B3 in the same appendix presents the standard deviations and correlations implied by the moment estimator of Σ in (17).

related with contemporaneous stock excess returns, but significantly predict positive future stock excess returns. As a result, the dividend-price ratio causes mean reversion in stock excess returns.

5.2 The Term Structure of Funding Spreads

As we saw above, our proposed method of optimizing an objective function in the log return on the funding ratio of assets to FRA liabilities automatically generates a term structure of funding spreads (6) evaluated at the optimal asset allocation, w_t^{Δ}. We can also manually calculate the funding spreads at the asset allocation outcome, w_t^0, from optimizing an objective function in the terminal log return on the funding ratio of assets to RF liabilities. By comparing the difference between the resulting term structures of funding spreads, Δ_t^s, and its components, π_t^s and λ_t^s, we can assess the funding risks implied by the two optimal portfolios. We will try this for different combinations of the parameters involved in the two objective functions: k, the investment horizon, ϕ, the relative consumption growth, γ, the coefficient of relative risk aversion, and F_t^0, the initial funding ratio. Recall that only the ratio of the initial funding ratio F_t^0 to the funding threshold τ is relevant for determining the underfunding probabilities and recovery fractions, not the absolute value of the two parameters. For this reason, we decided to set $\tau = 1$ and vary F_t^0.

Table 1 shows funding spreads, FRPs, underfunding probabilities and recovery fractions for a fixed maturity of one year (four quarters) for different values of k, ϕ, γ and F_t^0. What is clear is that, for the parameter constellations under consideration, the funding spreads resulting from using the asset allocation w_t^{Δ} are smaller than the funding spreads derived from w_t^0. The pension plan which determines the optimal asset allocation using FRA liabilities (i.e., w_t^{Δ}) will be less prone to underfunding and usually achieves higher recovery fractions than the plan using RF liabilities in (15).

The first panel of Table 1 shows the impact of varying the initial funding ratio on the funding spread for a given investment horizon of $k = 5$ years, relative consumption growth $\phi = 1.04$, and relative risk aversion $\gamma = 5$. F_t^0 takes values ranging between 0.75, which corresponds to a situation of severe underfunding relative to the threshold $\tau = 1$, and 1.0, which corresponds to a situation of full funding. It is not surprising that the funding spread decreases with increasing initial funding from $\Delta_t^4 = 0.208$ when $F_t^0 = 0.75$ to $\Delta_t^4 = 0.012$ when $F_t^0 = 1.0$. The appropriate funding spread for a pension plan with 100% initial funding reflects an underfunding probability of $\pi_t^4 = 0.1855$ and a recovery fraction of $\lambda_t^4 = 0.9446$. To assess the magnitude of these funding spreads, we compute the spreads between the yields on long-term corporate bonds with maturities of 20 years and above and the yield on a Treasury bond with a constant maturity of 10 years.[14] Depending on the credit rating of the issuing companies, we obtain the following average spreads from annual data between 1980 and 2006: 0.0105 (AAA), 0.0137 (AA), 0.0167 (A) and 0.0214 (BAA). Thus, the one-year spread for a 100%-funded pension plan approximately corresponds to the long-term average yield spread on AAA-rated corporate bonds. A 90%-funded pension plan would need a funding spread that is more than twice the average yield spread on BAA-rated bonds to appropriately reflect the risk underlying the promised pension payment at the one-year horizon.

The second panel of Table 1 shows the effect of varying the relative risk aversion on the funding spread for a given investment horizon of $k = 5$ years, relative consumption growth $\phi = 1.04$, and initial funding ratio $F_t^0 = 1.0$. As might be anticipated, higher relative risk aversion parameters imply lower funding spreads, mostly because underfunding probabilities are reduced as a result of the adoption of a more conservative asset allocation. A pension plan whose trustees have relative risk aversion $\gamma = 2$ should use a one-year funding spread of 0.041, which decreases to 0.012 for $\gamma = 5$ and to 0.005 for $\gamma = 10$.

The third panel of Table 1 shows the impact of a variation in relative consumption growth, ϕ, in the state of overfunding to the state of underfunding for given investment horizon of $k = 5$ years, relative risk aversion $\gamma = 5$ and initial funding ratio $F_t^0 = 1.0$. Recall that ϕ is the unknown parameter of the stochastic discount factor. Hence, a variation of ϕ directly affects the one-year FRP, θ_t^4.

[14] The corporate bond yields are derived from Moody's long-term corporate bond yield index available from Moody's website. Yields on Treasury bonds with a constant maturity of 10 years are available from the Federal Reserve statistical release (http://federalreserve.gov/releases/h15/data.htm).

Increasing the relative consumption growth from $\phi = 1.02$ (which is consistent with 4% consumption growth in the state of overfunding and 2% consumption growth in the state of underfunding) to $\phi = 1.06$ (which is consistent with 8% consumption growth in the state of overfunding and 2% consumption growth in the state of underfunding) increases the FRP from $\theta_t^4 = 0.001$ to $\theta_t^4 = 0.003$. The increase in the FRP leads to an increase in the funding spread by a corresponding amount. While the recovery fraction, λ_t^4, remains unchanged, the underfunding probability decreases from $\pi_t^4 = 0.185$ when $\theta_t^4 = 0.001$ to $\pi_t^4 = 0.183$ when $\theta_t^4 = 0.003$. This is a consequence of a small change in the optimal asset allocation caused by the variation of θ_t^4.

Finally, the fourth panel of Table 1 contains results from varying the investment horizon, k, for given relative consumption growth $\phi = 1.04$, relative risk aversion $\gamma = 5$ and initial funding ratio $F_t^0 = 1.0$. We observe that a pension plan with a longer investment horizon should discount one-year liabilities using larger funding spreads of 0.019 for k = 10 years, compared with 0.016 for k = 7.5 years and 0.012 for k = 5 years. This is because one-year underfunding probabilities increase and one-year recovery fractions decrease with an increasing investment horizon. The explanation for this is that the long-term investor attempts to exploit mean reversion in stock returns with a large holding in stocks, which favorably impacts the long-term Sharpe ratio for stocks, but has little effect at the one-year horizon. Thus, at the one-year horizon, the asset allocation of a long-term investor is actually quite risky and this is reflected in the higher funding spread.

Figures 1 and 2 present the term structure of funding spreads (6) for maturities of up to 10 years for a pension plan with an investment horizon corresponding to the liability horizon $k = 5$ and relative risk aversion of $\gamma = 5$. In Figure 1, we consider an initial funding ratio of $F_t^0 = 1.0$, while in Figure 2, the initial funding ratio is reduced to $F_t^0 = 0.75$. The upper graph in each figure shows the funding spread and FRP curves, while the lower graph displays the corresponding underfunding probabilities and recovery fractions. We present the spreads automatically generated from computing w_t^Δ and the spreads manually calculated from w_t^0. Although both funding spreads are close to each other, the spread generated from the portfolio w_t^0 always exceeds the spread related to w_t^{Δ}. Note that the figures show annualized spreads obtained by dividing the quarter s in equation (6) by four. FRPs are also annualized.

Figure 1 shows that funding spreads decline rapidly towards zero with increasing maturity. This is because the liabilities in our (somewhat artificial) example do not grow systematically over time and are only affected by interest rate risk, while the assets have positive expected returns. Thus, the probability of underfunding decreases with maturity. This can be seen clearly from the lower graph in Figure 1. Recovery fractions decrease with maturities below about 2.5 years and increase for higher maturities. The initial decrease is due to the volatility of the funding ratio return, while the subsequent increase is due to the effect of positive expected asset returns on raising the expected funding ratio and hence λ_t^s.

The funding spreads for the seriously underfunded pension plan in Figure 2 start at a much higher level than those for the fully funded pension plan in Figure 1, but decline even faster to zero with increasing maturity. This is because of underfunding probabilities, which are very close to unity for the early quarters (recall $F_t^0 = 0.75$ and $\tau = 1$), but subsequently decline rapidly to about zero at a horizon of 40 quarters as the expected funding ratios increase. The term structure of recovery fractions is upward sloping which can again be explained by the increase in expected funding ratios. The maximum recovery fraction in Figure 2 is only slightly higher than the minimum recovery fraction in Figure 1 reflecting the 30% difference in initial funding.

The FRP, θ_t^s, declines with maturity and increasing recovery fractions in Figure 1 (for $F_t^0 = 1.0$) from about 0.006 at the one-quarter horizon to zero at the four-year horizon. For all maturities, the risk premia in our example are too small to have a sizable impact on funding-risk spreads. From Figure 2 (for $F_t^0 = 0.75$), we see that the FRP first increases from zero to about 0.003 at the two-year horizon and then decreases again to zero at the seven-year horizon. This pattern can be explained using our comparative statics results for the FRP in Section 3.3. Recall that the FRP decreases with increasing maturity and an increasing recovery fraction and increases with a decreasing underfunding probability whenever $\phi^{-\gamma} < \lambda_t^s (\pi_t^s)^2 (1-\pi_t^s)^{-2}$. For maturities below two years, the latter effect dominates the effects of an increasing maturity and an increasing recovery fraction on the risk premium.

The existence of a term structure of funding spreads as shown in Figures 1 and 2 brings into question the current practice in the U.S. of using maturity-invariant discount rates for liability valuation purposes (often the yield on AA-rated long-term corporate bonds). A constant discount rate only can be justified in the unlikely case where the slope of the term structure of interest rates exactly offsets the slope of the term structure of funding spreads.

5.3 Optimal Asset Allocation

We compute the optimal asset allocations w_t^0 and w_t^Δ for a wide range of initial funding ratios between $F_t^0 = 0.025$ and $F_t^0 = 1.625$ for fixed $\tau = 1$ and $\phi = 1.04$ and a baseline parameter specification with investment horizon $k = 5$ and relative risk aversion $\gamma = 5$. In addition, we provide two comparative statics results for an increase in the relative risk aversion, to $\gamma = 10$, and in the investment horizon, to $k = 10$, leaving all other parameters unchanged. Figure 3 displays the optimized outcomes for the baseline specification. Specifically, the upper graphs in the figure contain plots of the optimal allocation to stocks and the one-year funding spread over the range of initial funding ratios, while the lower graphs contain plots of the one-year underfunding probability and the one-year recovery fraction over the range of initial funding ratios. We present results for both w_t^0 and w_t^Δ.

The optimal allocation to stocks in portfolio w_t^0 is independent of the initial funding ratio as predicted by Hoevenaars et al. (2008). In the upper graph in Figure 3, this allocation is represented by the horizontal line at a level of 73% stocks. By contrast, the optimal allocation to stocks in portfolio w_t^Δ depends on the initial funding ratio, since this affects both the underfunding probability (7) and the recovery fraction (8). From Figure 3, we see that the optimal allocation to stocks in portfolio w_t^Δ is a highly non-linear function of the initial funding ratio. We first describe the function and then provide an explanation for it.

The w_t^Δ function has four segments which correspond to four states of the initial funding ratio. State 1, which we define as a state of "critical underfunding", holds for initial funding ratios below 40% ($F_t^0 \leq 0.4$). In this segment, the allocation to stocks is both relatively low (around 25%) and invariant to the size of the initial funding ratio below the critical threshold. State 2, which we define as a state of "severe underfunding", holds for initial funding ratios between 40% and 75% ($0.40 < F_t^0 \leq 0.75$). The moment the initial funding ratio rises above 40% initial funding, the allocation to stocks jumps to a level of about 90%, which is above the optimal allocation to stocks in portfolio w_t^0 (around 73%). In this segment, the optimal allocation to stocks in portfolio w_t^Δ is a concave function of the initial funding ratio, always lying *above* the corresponding allocation in w_t^0, reaching a peak of 97% at an initial funding ratio of 50%, before falling back to 73% when the initial funding ratio reaches 75%. State 3, which we define as a state of "moderate underfunding", holds for initial funding ratios between 75% and 100% ($0.75 < F_t^0 \leq 1$). In this segment, w_t^Δ is a convex function of the initial funding ratio and always lies *below* w_t^0. The allocation to stocks reaches a local minimum in this state of approximately 67% at an initial funding ratio of 90%. State 4 is a state of "overfunding" ($F_t^0 > 1$). The optimal allocation to stocks in portfolio w_t^Δ is an increasing function of the initial funding ratio and rapidly approaches the optimal allocation to stocks in portfolio w_t^0. For initial funding ratios larger than 120%, the two allocations coincide exactly.

What determines this functional form for the optimal allocation to stocks in portfolio w_t^Δ? The lower graph in Figure 3 helps explain the optimal asset allocation in state 1 (critical underfunding). We see that the one-year underfunding probability equals unity below an initial funding threshold of 65%. Below 40%, all underfunding probabilities equal unity for horizons up to and including the maturity of the pension liability. In this state, the initial funding ratio is so low that the asset allocation cannot influence underfunding probabilities for nearby horizons. Thus, the asset allocation remains invariant to changes in the initial funding ratio. The funding spreads, reflecting the unit underfunding probabilities, are extremely high for short maturities. Figure 2 shows that the one-quarter funding spread for an initial funding ratio of 75% almost equals 2: for funding ratios below 40%, the spread is considerably higher. As a consequence, the FRA liability value at the investment horizon decreases and the expected log funding ratio return increases. For the given relative risk aversion, the pension plan compensates this increase by choosing an asset allocation with a relatively low stock exposure.

The lower graph in Figure 3 also helps explain the asset allocation in state 4 (overfunding). Since funding spreads rapidly approach zero for initial funding ratios in excess of 100%, the stock allocation in portfolio w_t^{Δ} converges to that in portfolio w_t^0. This is because the underlying objective functions only differ by the presence of funding spreads.

The asset allocations associated with states 1 and 4 could be readily predicted. More interesting, because they are less predictable, are the asset allocations associated with states 2 and 3. In state 2 (severe underfunding), the initial funding ratio is sufficiently large that a change in the asset allocation has an impact on underfunding probabilities for longer horizons. In this state, the pension plan bets on the equity premium in order to increase the expected funding ratio at the investment horizon. This results in a higher stock weighting for w_t^{Δ} compared with w_t^0. If a pension plan finds itself in state 3 (moderate underfunding), it wants to reduce the risk of making the deficit even worse. It therefore takes a more cautious approach than it would in state 2 and chooses a lower stock weighting than it would in that state, one which is even lower than the stock weighting in w_t^0.

Either increasing the coefficient of relative risk aversion or increasing the investment horizon changes the optimal asset allocation, but does not alter the general pattern outlined above.[15] In line with earlier findings by Campbell et al. (2003), Campbell and Viceira (2005), and Hoevenaars et al. (2008), higher risk aversion implies lower stock holdings, while a longer investment horizon leads to higher stock exposure as a consequence of mean reversion in stock returns.

However, funding spreads, underfunding probabilities and recovery fractions are not very sensitive to the size of the coefficient of relative risk aversion or the length of the investment horizon. One-year

[15] Figures C1 and C2 in appendix C show the comparative statics results from increasing $\gamma = 5$ to $\gamma = 10$ and from increasing $k = 5$ to $k = 10$, respectively. The four states identified in Figure 3 are clearly discernible in these figures too, although the corresponding initial funding ratio boundaries differ slightly.

funding spreads decrease from a level in excess of 3.5 at $F_t^0 = 0.2$ to below 0.1 at an initial funding ratio of around 80%. A closer look at the data reveals that the funding spreads for w_t^Δ slightly exceed those for w_t^0 in state 1. Since one-year underfunding probabilities for w_t^0 and w_t^Δ in this state both equal zero, the difference in funding spreads is explained by one-year recovery fractions which are slightly higher for w_t^0 than for w_t^Δ. However, in states 2 to 4, a pension plan using the proposed asset allocation w_t^Δ will always exhibit a smaller funding spread than a pension plan using w_t^0.

Table 2 sheds a more quantitative light on the asset allocation "wave" in states 2 to 4 than can be gleaned from Figure 3 alone.[16] Table 2 shows the optimal allocations to cash, bonds, and stocks in portfolios w_t^0 and w_t^Δ. We present results from the optimization exercise for fixed $\tau = 1$ and $\phi = 1.04$ and for a range of initial funding ratios, $F_t^0 \in (0.75, 0.90, 1.00, 1.10)$, falling within states 2 to 4 for our baseline parameter specification, $k = 5$ and $\gamma = 5$, together with comparative statics results for $\gamma = 10$ and $k = 10$. We can see from Table 2 that a 100% funded pension plan, which adopts the optimal portfolio for FRA liabilities, w_t^Δ, would allocate 69.1% of its assets to stocks in our baseline parameter specification. A pension plan selecting portfolio w_t^0, which is optimal for RF liabilities, would allocate 73.4% to stocks. From Table 1, we know that this stock exposure translates into a one-year funding spread of 0.012 for w_t^Δ and 0.014 for w_t^0, because the underfunding probability is smaller and the recovery fraction is larger for w_t^Δ. The 4.3 percentage point difference between the two stock allocations is distributed to cash (2.6) and bonds (1.7) in portfolio w_t^Δ.

Increasing the coefficient of relative risk aversion to $\gamma = 10$ decreases the optimal stock allocations to 40.6% in portfolio w_t^Δ and to 41.7% in portfolio w_t^0. On the other hand, increasing the investment horizon to twice the maturity of the liabilities, $k = 10$, increases stock allocations to 89.1% (w_t^Δ) and 93.0% (w_t^0), respectively.

[16] Or Figures C1 and C2 in appendix C.

6. Conclusion

In this paper, we critically review the current practice of pension liability valuation using discount rates which do not reflect the risk inherent in the pension promise. We propose a new approach to the valuation of pension obligations which depends on the term structure of funding-risk-adjusted discount rates that, in turn, depend on the asset allocation of the pension fund. When the asset allocation is based on an objective function in assets and funding-risk-adjusted liabilities, discount rates and portfolio weights are interdependent and can be determined in a single optimization step. We demonstrate this for a pension plan which optimizes the expected utility in a terminal funding ratio (of assets to funding-risk-adjusted liabilities) at a finite horizon. Since our approach does not require the pension plan to make *any* assumptions about the chosen discount rate for liability valuation purposes, we effectively remove an important degree of freedom from the plan sponsor's set of choices on how to measure liabilities and, hence, reduce the likelihood of the liabilities being reported in a strategic way to satisfy goals that might conflict with the proper objective of managing the plan in the best interests of the plan's beneficiaries. On the contrary, we believe that our proposed valuation method has advantages for all stakeholders of the pension plan, including the sponsoring company and its shareholders, and this is because it increases transparency with respect to the plan's expected future funding position.

On the basis of applying our model to U.S. data, we can conclude that a pension plan with an initial funding ratio that is not critically low and which maximizes our proposed objective function will always exhibit lower underfunding probabilities, higher recovery fractions, and lower funding spreads than a pension plan which maximizes a conventional objective function in the log return of assets over risk-free liabilities. The funding spreads also vary with maturity, which brings into question the current practice of using constant discount rates. The optimal asset allocation varies with the initial funding ratio of the pension plan. A plan which follows our methodology will always allocate a smaller weighting of plan assets to stocks than is conventional, except when the initial funding ratio is severe (a state of underfunding which lies between critical and moderate). A severely underfunded pension plan tries to benefit from the equity premium, while a moderately underfunded pension plan takes a more cautious approach and tries to avoid a further reduction in the funding ratio by choosing a lower stock exposure. A critically underfunded plan faces such high funding risk that the optimal allocation to stocks (which is less than half the conventional level in our example) is independent of the funding ratio.

Finally, we note that our methodology is very easy to implement and converges very quickly. We would therefore expect pension plan stakeholders and pension regulators to show interest in the funding-risk-adjusted measure of the pension liabilities of private companies in due course.

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Tables and Figures

Table 1: One-year funding spreads, funding-risk premia, underfunding probabilities and recovery fractions

				Δ_t^4		θ_t^4		π_t^4		λ_t^4	
k	ϕ	γ	F_t^0	w_t^0	w_t^Δ	w_t^0	w_t^Δ	w_t^0	w_t^Δ	w_t^0	w_t^Δ
5	1.04	5	0.75	0.208	0.208	0.002	0.002	0.956	0.956	0.822	0.822
			0.90	0.053	0.050	0.004	0.004	0.530	0.545	0.914	0.920
			1.00	0.014	0.012	0.002	0.002	0.194	0.184	0.941	0.945
5	1.04	2	1.00	0.047	0.041	0.003	0.002	0.302	0.291	0.859	0.871
		5		0.014	0.012	0.002	0.002	0.194	0.184	0.941	0.945
		10		0.005	0.005	0.001	0.001	0.111	0.107	0.969	0.970
5	1.02	5	1.00	0.013	0.011	0.001	0.001	0.194	0.185	0.941	0.945
	1.04			0.014	0.012	0.002	0.002	0.194	0.184	0.941	0.945
	1.06			0.015	0.013	0.003	0.003	0.194	0.183	0.941	0.945
5	1.04	5	1.00	0.014	0.012	0.002	0.002	0.194	0.184	0.941	0.945
7.5				0.017	0.016	0.002	0.002	0.212	0.205	0.932	0.936
10				0.020	0.019	0.003	0.003	0.226	0.220	0.924	0.928

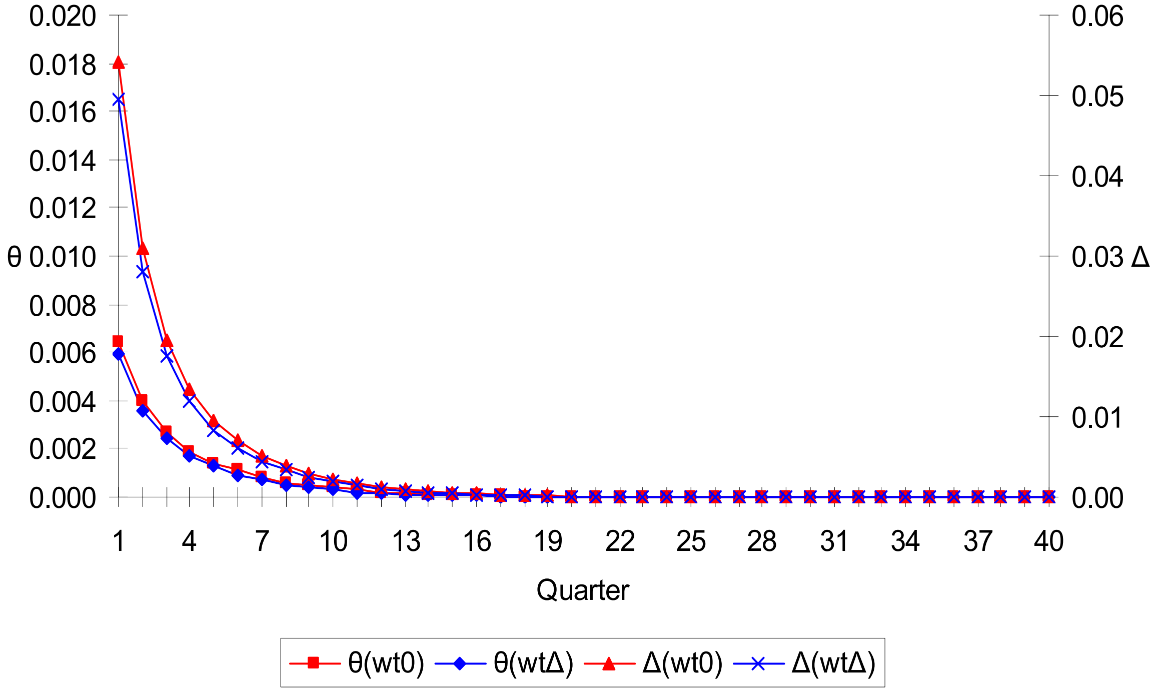
Note: k is the investment horizon in years, ϕ relative consumption growth, γ the coefficient of relative risk aversion and F_t^0 the initial funding ratio (assuming $\tau = 1$). Δ_t^4 denotes the one-year (four-quarter) funding spread, θ_t^4 the one-year funding-risk premium, π_t^4 the one-year underfunding probability and λ_t^4 the one-year recovery fraction. These are valued at w_t^0 and w_t^Δ, the optimal portfolios for RF and FRA liabilities, respectively.

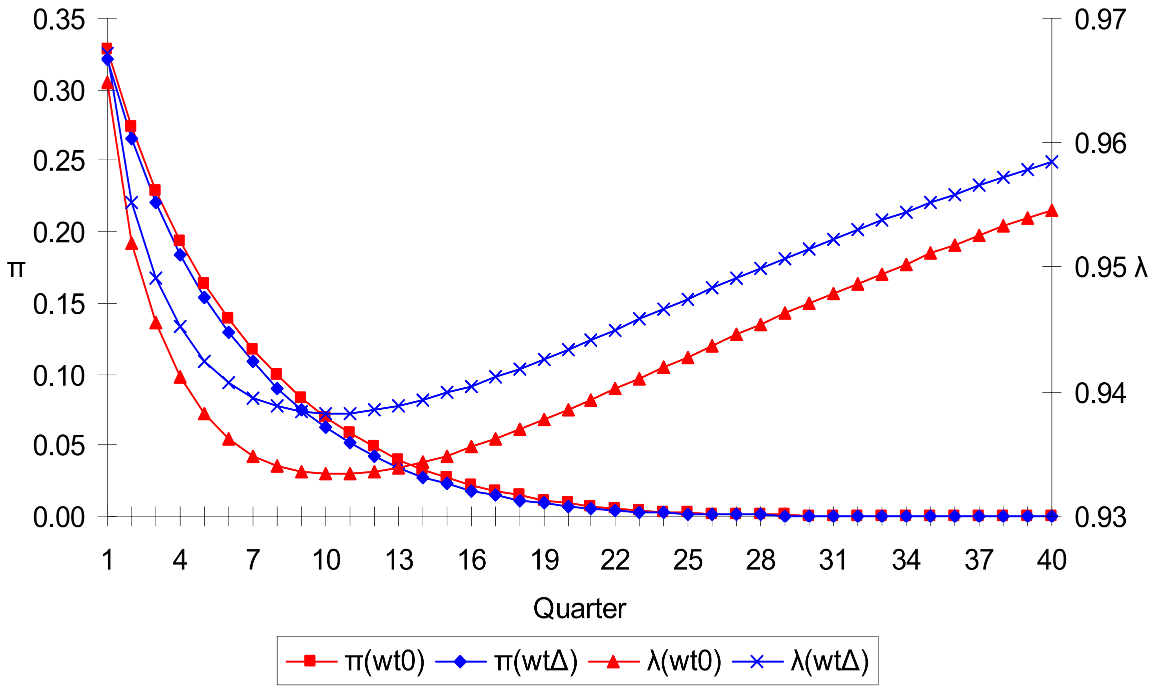
Table 2: Optimal asset allocation

			Cash		Bonds		Stocks	
k	γ	F_t^0	w_t^0	w_t^Δ	w_t^0	w_t^Δ	w_t^0	w_t^Δ
5	5	0.75	-0.024	-0.045	0.290	0.307	0.734	0.738
		0.90	-0.024	0.013	0.290	0.317	0.734	0.669
		1.00	-0.024	0.002	0.290	0.307	0.734	0.691
		1.10	-0.024	-0.013	0.290	0.297	0.734	0.717
5	10	0.75	0.235	0.203	0.348	0.352	0.417	0.445
		0.90	0.235	0.247	0.348	0.359	0.417	0.394
		1.00	0.235	0.241	0.348	0.353	0.417	0.406
		1.10	0.235	0.235	0.348	0.348	0.417	0.416
10	5	0.75	-0.145	-0.139	0.215	0.240	0.930	0.899
		0.90	-0.145	-0.120	0.215	0.245	0.930	0.875
		1.00	-0.145	-0.126	0.215	0.235	0.930	0.891
		1.10	-0.145	-0.134	0.215	0.226	0.930	0.908

Note: k is the investment horizon in years, γ the coefficient of relative risk aversion and F_t^0 the initial funding ratio (assuming $\tau = 1$). Cash, bonds and stocks refer to the allocations to these asset classes. These are valued at w_t^0 and w_t^Δ, the optimal portfolios for RF and FRA liabilities, respectively. The parameter of relative consumption growth is set to $\phi = 1.04$.

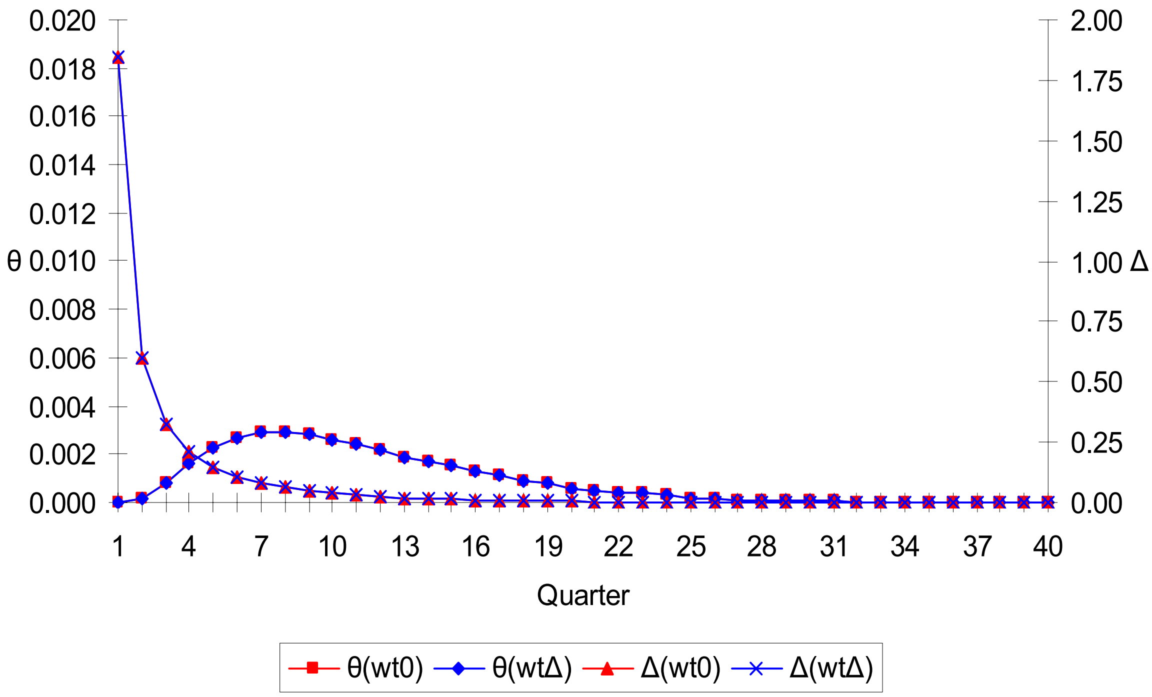
Figure 1: Term structure of funding spreads, funding-risk premia, underfunding probabilities, and recovery fractions for $k = 5$ years, $\phi = 1.04$, $\gamma = 5$, $\tau = 1$, $F_t^0 = 1.0$

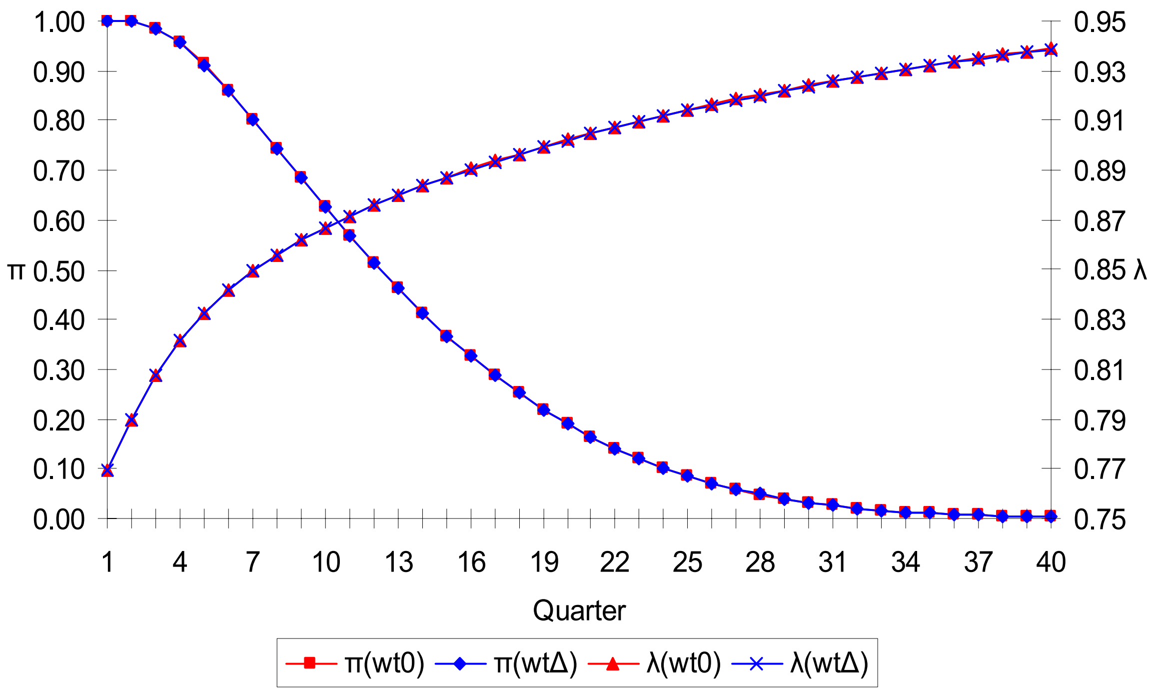




Note: The graphs show annualized funding spreads, annualized funding-risk premia, underfunding probabilities, and recovery fractions over 40 quarters. These are valued at w_t^0 and w_t^Δ, the optimal portfolios for RF and FRA liabilities, respectively.

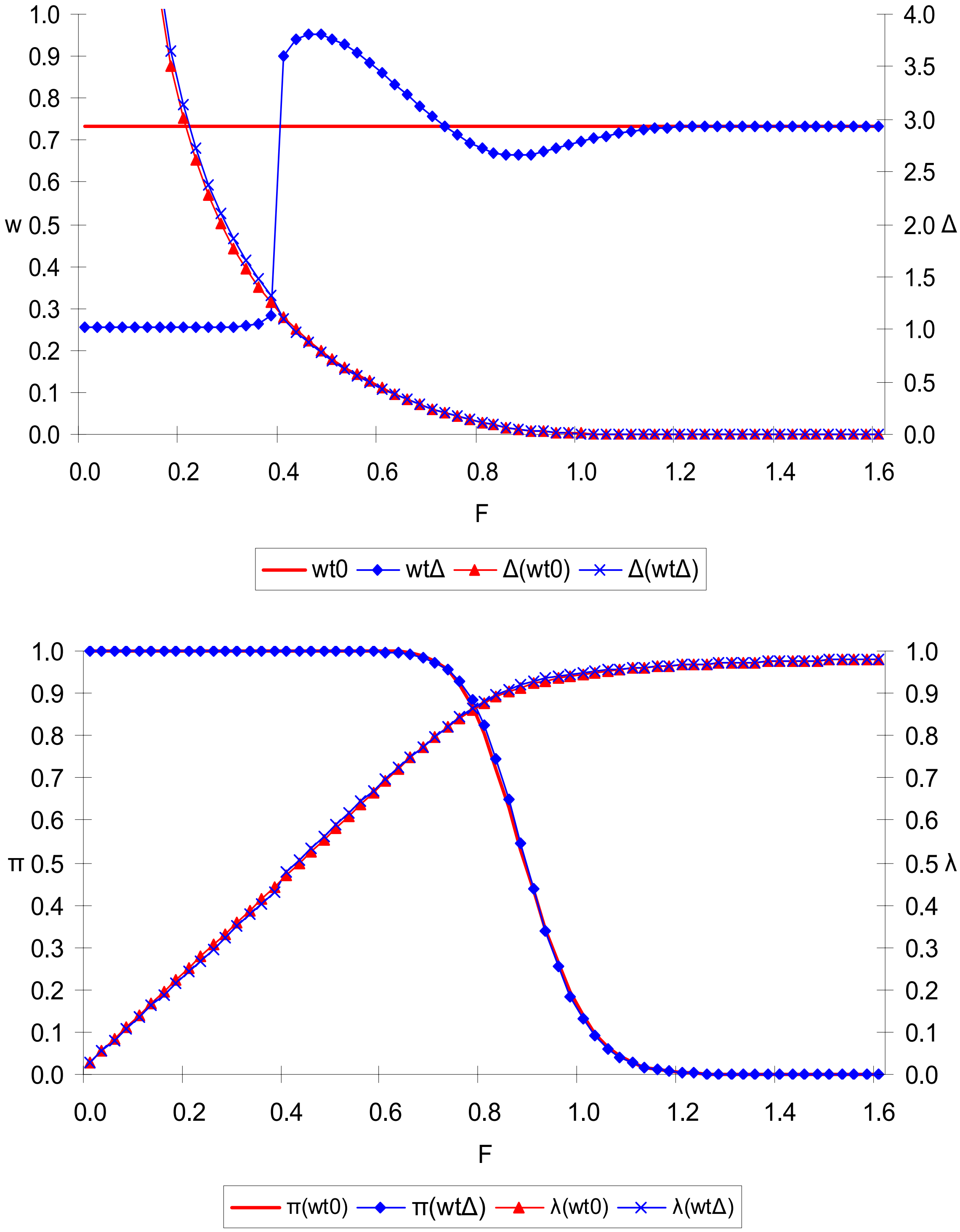
Figure 2: Term structure of funding spreads, funding-risk premia, underfunding probabilities, and recovery fractions for $k = 5$ years, $\phi = 1.04$, $\gamma = 5$, $\tau = 1$, $F_t^0 = 0.75$





Note: As per Figure 1.

Figure 3: Stock allocations, funding spreads, underfunding probabilities and recovery fractions as a function of the initial funding ratio for $k = 5$ years, $\phi = 1.04$, $\gamma = 5$, $\tau = 1$



Note: The graphs show the optimal stock allocations and annualized funding spreads (up to a ceiling of 4.0), underfunding probabilities and recovery fractions at 1-year maturity. w_t^0 and w_t^Δ are the optimal portfolios for RF and FRA liabilities, respectively.

Appendix A: Derivations

This appendix derives the cumulative log funding ratio returns $s^0_{t+k} = \ln\left(R^A_{t+k} / R^{L,0}_{t+k}\right)$ and $s^\Delta_{t+k} = \ln\left(R^A_{t+k} / R^{L,\Delta}_{t+k}\right)$. Using the now familiar approximation of the log portfolio return introduced by Campbell and Viceira (2002), the log of the cumulative portfolio return, R^A_{t+k}, can be written as[17]

$$r^A_{t+k} = \ln R^A_{t+k} = r^f_{t+k} + w_t' r^e_{t+k} + 0.5 w_t' \text{diag}\left(V_t\left[r^e_{t+k}\right]\right) - 0.5 w_t' V_t\left[r^e_{t+k}\right] w_t \quad \text{(A1)}$$

with $r^f_{t+k} = \ln R^f_{t+k}$ and $r^e_{t+k} = \ln R_{t+k} - \ln R^f_{t+k}$. Lower-case letters denote variables in logs (unless defined otherwise) and $V_t[\cdot]$ is the conditional variance-covariance matrix operator at time t, with $\text{diag}(V_t[\cdot])$ as a vector of the main diagonal elements of $V_t[\cdot]$. Using the risk-free rate, we can re-write the cumulative liability return for $s = k$ in (14) as $R^L_{t+k} = v_t' R_{t+k} = R^f_{t+k} + v_t'\left(R_{t+k} - R^f_{t+k}\right)$ in order to apply again a log-linearization with $r_{t+k} = \ln R_{t+k}$

$$r^L_{t+k} = \ln R^L_{t+k} = v_t' r_{t+k} + 0.5 v_t' \text{diag}\left(V_t\left[r_{t+k} - r^f_{t+k}\right]\right) - 0.5 v_t' V_t\left[r_{t+k} - r^f_{t+k}\right] v_t \,. \quad \text{(A2)}$$

We denote the log liability return as $r^{L,0}_{t+k}$ if based on RF liabilities and as $r^{L,\Delta}_{t+k}$ if based on FRA liabilities.[18] The log funding ratio returns follow immediately from (A1) and (A2).

Under the assumption that the log funding ratio return is normally distributed, we can express the conditional underfunding probability, π^s_t, in (7) and the recovery fraction, λ^s_t, in (8) using moments of s^0_{t+s}. The log funding ratio at time $t+s$ using RF liabilities is distributed as

$$\ln\left(F^0_{t+s}\right) = \ln\left(F^0_t \frac{R^A_{t+s}}{R^{L,0}_{t+s}}\right) = f^0_t + s^0_{t+s} \sim N\left(f^0_t + E_t\left[s^0_{t+s}\right], V_t\left[s^0_{t+s}\right]\right) \quad \text{(A3)}$$

where $f^0_t = \ln\left(F^0_t\right)$ is the log of the initial funding ratio. From (A3), we obtain

[17] The approximation is based on a second-order Taylor series expansion and is needed because the log portfolio return is not equal to the portfolio-weighted average of log returns.

[18] We ignore in the Taylor series expansion for the latter the fact that v_t depends on r^A_{t+k}-r^f_{t+k} as well. This is justified by the fact that since the elements of v_t sum up to unity, the impact of this simplification is negligible.

$$\pi_t^s = \Pr_t\left(\ln\left(\frac{A_{t+s}}{L_{t+s}^0}\right) < \ln(\tau)\right) = \Pr_t\left(f_t^0 + s_{t+s}^0 < \ln(\tau)\right) = \Phi\left(\frac{\ln(\tau) - \left(f_t^0 + E_t\left[s_{t+s}^0\right]\right)}{V_t^{0.5}\left[s_{t+s}^0\right]}\right) \tag{A4}$$

$$\lambda_t^s = \frac{1}{\tau\pi_t^s}\exp\left(f_t^0 + E_t\left[s_{t+s}^0\right] + 0.5V_t\left[s_{t+s}^0\right]\right)\cdot\Phi\left(\frac{\ln(\tau) - \left(f_t^0 + E_t\left[s_{t+s}^0\right]\right) - V_t\left[s_{t+s}^0\right]}{V_t^{0.5}\left[s_{t+s}^0\right]}\right) \tag{A5}$$

for (7) and (8) where $\Phi(\cdot)$ denotes the c.d.f. of the standard normal distribution. Equation (A5) exploits properties of the truncated lognormal distribution (see Lien, 1985). It is evident from (A4) and (A5) that only the ratio of the initial funding ratio, F_t^0, to the underfunding threshold, τ, is relevant for determining the underfunding probabilities and recovery fractions, not their absolute value. Thus, for given moments of the log funding ratio return s_{t+s}^0, the parameter constellation $F_t^0 = 1$ and $\tau = 0.8$ will give the same values for π_t^s and λ_t^s as the constellation $F_t^0 = 1.25$ and $\tau = 1.0$.

If the regulatory authority obliges the sponsoring company to close any funding gap, the funding threshold could be modeled as $\tau = 1 - N_{t+s}/L_{t+s}^0$ such that $\pi_t^s = \Pr_t\left((A_{t+s} + N_{t+s})/L_{t+s}^0 < 1\right)$ and $\lambda_t^s = E_t\left[(A_{t+s} + N_{t+s})/L_{t+s}^0 \mid (A_{t+s} + N_{t+s})/L_{t+s}^0 < 1\right]$ where N_{t+s} denotes the future net worth of the sponsoring company. To derive expressions like (A4) and (A5) for this case, assume that the net worth of the plan sponsor in period $t+s$ can be written as $N_{t+s} = N_t R_{t+s}^N$. Define the sum of assets of the pension plan and the net worth of the sponsoring company as $V_t = A_t + N_t$ (total assets) and the funding ratio in terms of total assets as $G_t^0 = (A_t + N_t)/L_t^0$. Let $a_t = A_t/(A_t + N_t)$ denote the fraction of the plan assets in total assets, V_t. Then the cumulative s-period return on total assets results as

$$R_{t+s}^V = \frac{V_{t+s}}{V_t} = \frac{A_{t+s} + N_{t+s}}{A_t + N_t} = \frac{A_t R_{t+s}^A + N_t R_{t+s}^N}{A_t + N_t} = R_{t+s}^N + a_t\left(R_{t+s}^A - R_{t+s}^N\right). \tag{A6}$$

Using again the Campbell and Viceira (2002) approximation of the log portfolio return, we obtain $r_{t+s}^V = r_{t+s}^N + a_t\left(r_{t+s}^A - r_{t+s}^N\right) + 0.5a_t(1-a_t)V_t\left[r_{t+s}^A - r_{t+s}^N\right]$. Assuming that the difference between the log return on total assets and the log return on the RF liabilities, $x_{t+s}^0 = r_{t+s}^V - r_{t+s}^{L,0}$, is normally distributed and denoting the log initial funding ratio as $g_t^0 = \ln G_t^0$, we find

$$\ln\left(\frac{A_{t+s}+N_{t+s}}{L^{0}_{t+s}}\right)=\ln\left(G^{0}_{t}\frac{R^{V}_{t+s}}{R^{L,0}_{t+s}}\right)=g^{0}_{t}+x^{0}_{t+s}\sim N\left(g^{0}_{t}+E_{t}\left[x^{0}_{t+s}\right],V_{t}\left[x^{0}_{t+s}\right]\right). \tag{A7}$$

From here we obtain the expressions of interest similar to equations (A4) and (A5)

$$\pi^{s}_{t}=\Pr_{t}\left(\ln\left(\frac{A_{t+s}+N_{t+s}}{L^{0}_{t+s}}\right)<0\right)=\Pr_{t}\left(g^{0}_{t}+x^{0}_{t+s}<0\right)=\Phi\left(-\frac{\left(g^{0}_{t}+E_{t}\left[x^{0}_{t+s}\right]\right)}{V^{0.5}_{t}\left[x^{0}_{t+s}\right]}\right) \tag{A8}$$

$$\lambda^{s}_{t}=\frac{1}{\pi^{s}_{t}}\exp\left(g^{0}_{t}+E_{t}\left[x^{0}_{t+s}\right]+0.5V_{t}\left[x^{0}_{t+s}\right]\right)\cdot\Phi\left(\frac{-\left(g^{0}_{t}+E_{t}\left[x^{0}_{t+s}\right]\right)-V_{t}\left[x^{0}_{t+s}\right]}{V^{0.5}_{t}\left[x^{0}_{t+s}\right]}\right). \tag{A9}$$

Appendix B: Tables

Table B1: Descriptive statistics

	Mean	St.dev.
Nominal T-bill return	5.21	1.41
Bond excess return	1.53	9.78
Stock excess return	7.05	15.74
Dividend-price ratio	-3.47	0.40
1-yr zero-coupon bond yield	5.56	2.92
2-yr zero-coupon bond yield	5.76	2.88
3-yr zero-coupon bond yield	5.93	2.81
4-yr zero-coupon bond yield	6.06	2.78
5-yr zero-coupon bond yield	6.14	2.74
1-yr zero-coupon bond return	-0.03	1.75
2-yr zero-coupon bond return	-0.04	3.15
3-yr zero-coupon bond return	-0.04	4.29
4-yr zero-coupon bond return	-0.03	5.32
5-yr zero-coupon bond return	-0.01	6.24

Note: With the exception of the dividend-price ratio, all statistics are annualized percentages. Mean log returns are adjusted by half of their variance to reflect mean gross returns. Quarterly data from 1952:Q2 –2005:Q4.

Table B2: VAR(1) estimation results

	Const.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	R^2
(1) Nominal T-bill return	0.00	0.97	0.01	0.00	0.00	-0.13	0.03	0.05	-0.05	-0.01	0.94
	0.65	*54.40*	*1.33*	*2.23*	*0.38*	*-2.25*	*0.59*	*1.16*	*-1.77*	*-0.30*	*0.00*
(2) Bond excess return	-0.01	0.02	-0.02	-0.06	-0.01	-1.58	0.20	-1.89	2.79	-0.84	0.09
	-0.42	*0.04*	*-0.12*	*-1.39*	*-0.57*	*-0.97*	*0.13*	*-1.48*	*3.31*	*-1.18*	*0.02*
(3) Stock excess return	0.17	-2.06	-0.16	0.07	0.04	-2.57	4.11	-4.93	2.43	0.56	0.11
	3.30	*-2.60*	*-0.51*	*0.99*	*2.79*	*-1.00*	*1.76*	*-2.44*	*1.81*	*0.49*	*0.00*
(4) Dividend-price ratio	-0.13	1.34	0.21	-0.06	0.97	2.77	-4.22	4.46	-2.24	-0.49	0.96
	-2.36	*1.65*	*0.64*	*-0.84*	*68.49*	*1.05*	*-1.76*	*2.15*	*-1.63*	*-0.42*	*0.00*
(5) 1-yr zero-coupon bond return	0.00	0.16	-0.03	-0.01	0.00	-0.37	-0.11	-0.17	0.43	-0.07	0.13
	-0.59	*1.81*	*-0.96*	*-1.91*	*-0.30*	*-1.31*	*-0.41*	*-0.78*	*2.90*	*-0.55*	*0.00*
(6) 2-yr zero-coupon bond return	-0.01	0.22	-0.05	-0.03	0.00	-0.21	-0.46	-0.32	0.72	-0.08	0.10
	-0.54	*1.41*	*-0.78*	*-1.92*	*-0.33*	*-0.41*	*-0.97*	*-0.79*	*2.66*	*-0.37*	*0.01*
(7) 3-yr zero-coupon bond return	-0.01	0.24	-0.03	-0.04	0.00	-0.47	0.03	-1.01	1.18	-0.24	0.11
	-0.52	*1.11*	*-0.31*	*-1.92*	*-0.36*	*-0.66*	*0.05*	*-1.83*	*3.24*	*-0.76*	*0.01*
(8) 4-yr zero-coupon bond return	-0.01	0.25	-0.03	-0.05	0.00	-1.10	0.34	-0.73	1.01	-0.28	0.09
	-0.50	*0.91*	*-0.28*	*-2.02*	*-0.38*	*-1.24*	*0.42*	*-1.05*	*2.19*	*-0.72*	*0.03*
(9) 5-yr zero-coupon bond return	-0.01	0.26	-0.01	-0.05	0.00	-0.94	0.09	-0.88	1.68	-0.71	0.09
	-0.52	*0.81*	*-0.06*	*-1.94*	*-0.42*	*-0.91*	*0.10*	*-1.08*	*3.13*	*-1.55*	*0.02*

Note: Column labeled 'const.' contains the OLS estimate of the intercept vector Φ_0, while columns (1) to (9) contain the OLS estimate of the matrix Φ_1, which contains the slope parameters for the vector of lagged dependent variables. The numbers in italics in the first 10 columns are t-values. The final column contains R^2 and (in italics) p-values from an F-test of joint significance for each equation. All variables are in logs. Q = 214 quarterly observations.

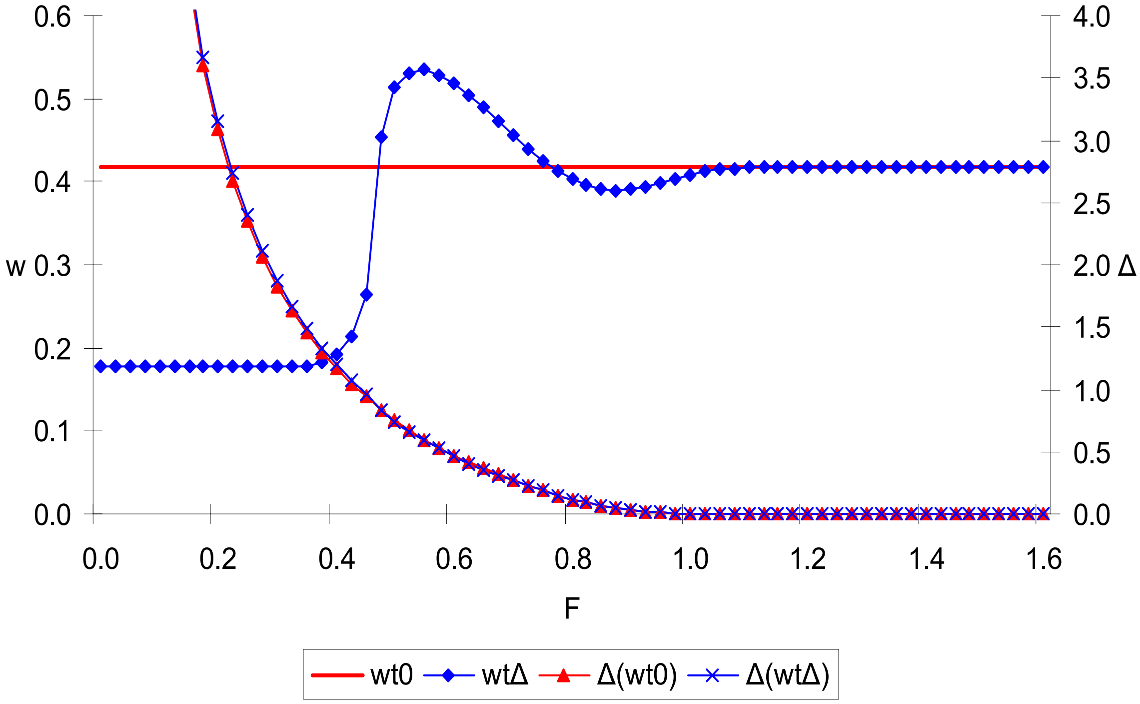
Table B3: Residual correlation matrix

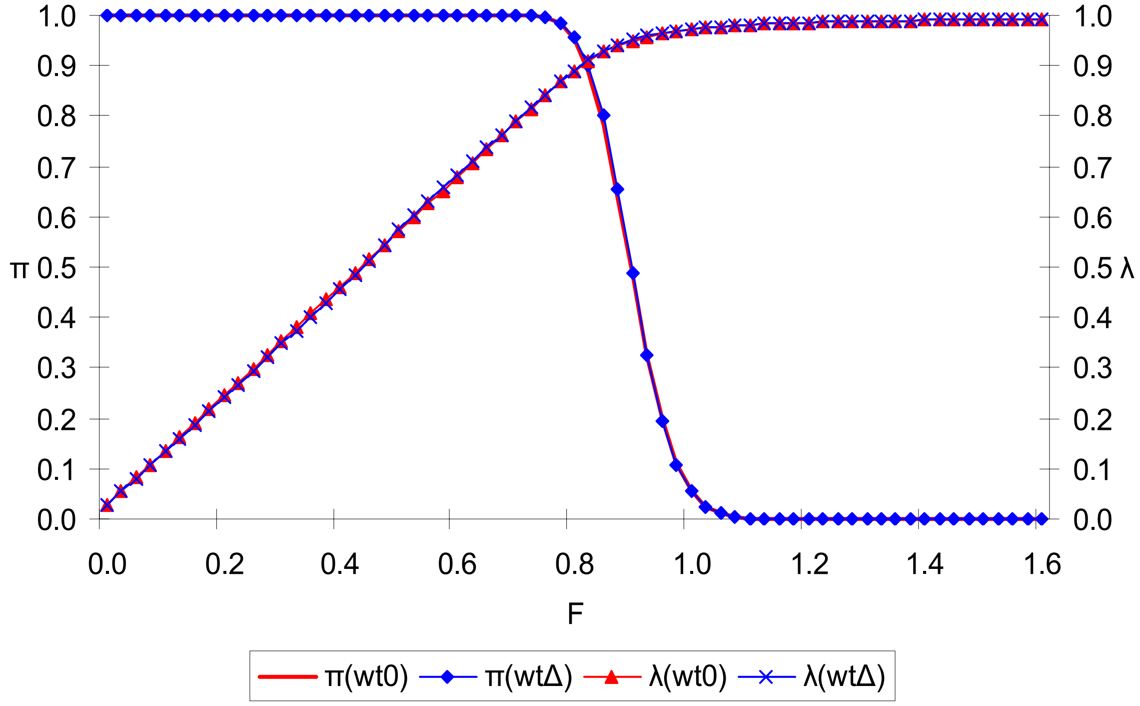
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) Nominal T-bill return	0.17	-0.50	-0.17	0.18	-0.78	-0.71	-0.64	-0.61	-0.57
(2) Bond excess return		4.67	0.19	-0.20	0.76	0.85	0.88	0.91	0.93
(3) Stock excess return			7.41	-0.98	0.16	0.14	0.12	0.13	0.11
(4) Dividend-price ratio				7.59	-0.17	-0.16	-0.14	-0.14	-0.13
(5) 1-yr zero-coupon bond return					0.82	0.96	0.93	0.89	0.87
(6) 2-yr zero-coupon bond return						1.49	0.98	0.96	0.95
(7) 3-yr zero-coupon bond return							2.03	0.99	0.98
(8) 4-yr zero-coupon bond return								2.54	0.99
(9) 5-yr zero-coupon bond return									2.97

Note: Correlation matrix implied by the estimated residual variance-covariance matrix Σ. Main diagonal elements contain standard deviations of quarterly residuals in %.

Appendix C: Figures

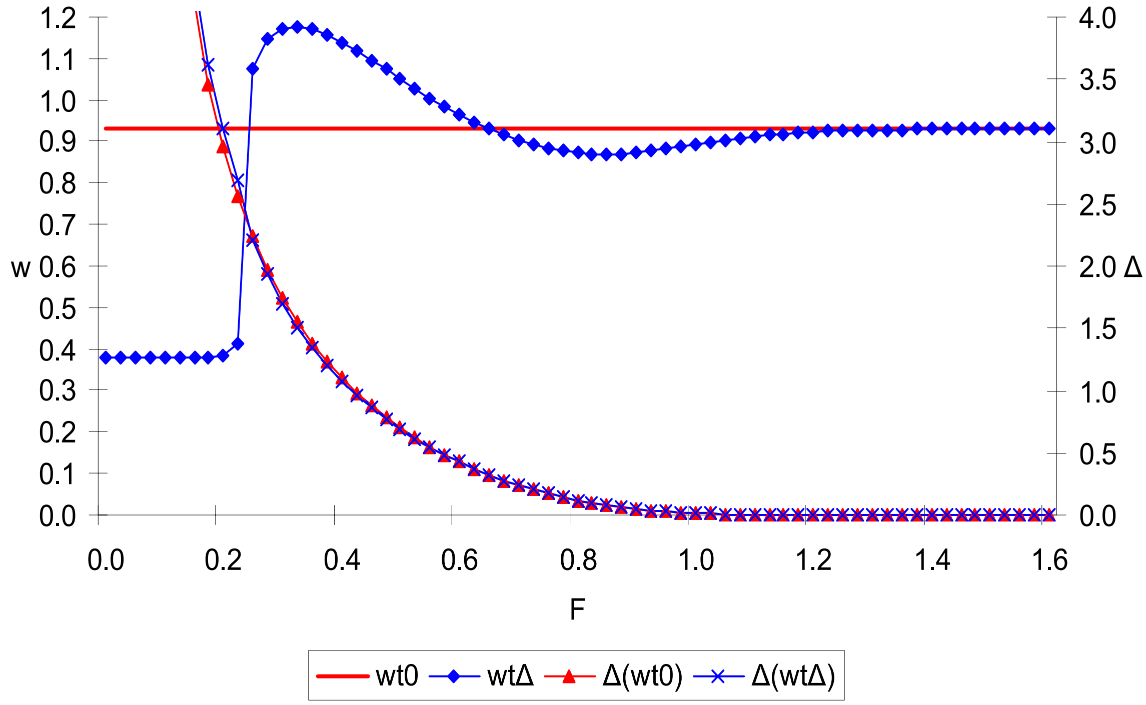
Figure C1: Stock allocations, funding spreads, underfunding probabilities and recovery fractions as a function of the initial funding ratio for $k = 5$ years, $\phi = 1.04$, $\gamma = 10$, $\tau = 1$

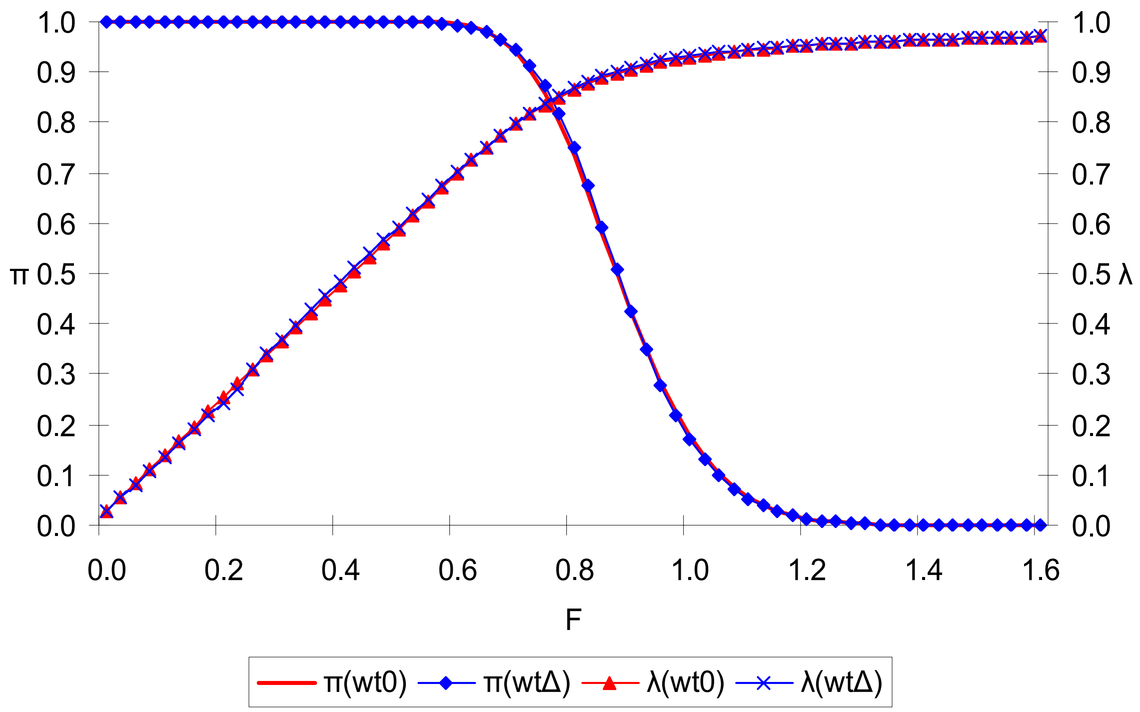




Note: As per Figure 3.

Figure C2: Stock allocations, funding spreads, underfunding probabilities and recovery fractions as a function of the initial funding ratio for $k = 10$ years, $\phi = 1.04$, $\gamma = 5$, $\tau = 1$





Note: As per Figure 3.