# Optimal Tax-Timing and Asset Allocation when Tax Rebates on Capital Losses are Limited*

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**Abstract**

This article analyzes the optimal dynamic consumption portfolio problem in the presence of capital gains taxes. It explicitly takes into account limited capital loss deduction and the 3,000 dollar amount that can be offset against other income. Constantinides (1983) shows that in tax-systems where capital gains and losses are subject to the same taxable treatment, it is is optimal to realize capital losses immediately. We generalize this finding to tax-systems where limited amounts of losses can be used against other income. In such tax-systems, the investment decision becomes substantially more difficult for two reasons. First, the investor has the opportunity of offsetting limited amounts of losses against other income. Hence, she has to make a decision on how to use a loss, i.e. whether to offset it against realized capital gains or to potentially postpone the realization of capital gains and offset it against other income. Second, because the tax rate on other income usually exceeds that on capital gains, in our setting it can be optimal to realize capital gains immediately, which prevents investors from getting locked in and helps to keep portfolios diversified.

**JEL Classification Codes:** G11, H24

**Key Words:** tax-timing, asset allocation, capital losses, tax loss carry-forward, limits on tax rebates

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# 1 Introduction

According to the seminal work of Constantinides (1983), it is optimal to realize losses immediately, and the tax realization strategy for an individual portfolio is separable from other aspects of portfolio choices under certain conditions. These include: (1) investors do not face any short-selling constraints, (2) wash sales are permitted,[1] (3) long-term and short-term capital gains are subject to the same capital gains tax rate, and (4) there is no limit on tax rebates for incurred capital losses.

There is an extensive body of literature studying optimal portfolio decisions that relaxes some of these assumptions. Relaxing assumption (1), Dybvig and Koo (1996) and DeMiguel and Uppal (2005) show that for short-selling constrained investors, optimal asset allocation decisions depend on the tax basis of the asset in a complicated fashion. Dammon et al. (2001) show that for short-selling constrained investors the diversification benefit of reducing a volatile position can significantly outweigh the tax cost of selling an asset with an unrealized capital gain.

Relaxing condition (2), Stiglitz (1983) suggests selling or shorting highly correlated assets instead of realizing capital gains to circumvent wash sale rules. Gallmeyer et al. (2006) address this issue in a multi-asset setting.

Eliminating assumption (3), Constantinides (1984) shows that it can be optimal to sell assets with an unrealized capital gain as soon as they qualify for long-term treatment in order to regain the opportunity to produce short-term losses.

This paper relaxes assumption (4) and studies optimal portfolio decisions when amounts of capital losses deductible against other income are limited. There are only two papers circulating that take the different taxable treatment of capital gains and losses explicitly into account. Gallmeyer and Srivastava (2003) deal with arbitrage concerns and show that under quite mild conditions, the lack of pre-tax arbitrage implies the lack of post-tax arbitrage. Ehling et al. (2008) deal with optimal investment decisions of private investors in tax-systems with no tax-rebate payments. While in their studies, losses can only be offset against capital gains, we allow for the opportunity of offsetting losses not exceeding $ 3,000 per year against other income.

This paper generalizes a key result of Constantinides (1983) by showing that it remains

[1] A transaction is termed a wash sale if a stock is sold to realize a capital loss and repurchased immediately. Under current tax-rules, wash sales do not qualify for the capital loss deduction if the same stock is repurchased within thirty days before or after the sale. Under current tax-law wash sales are forbidden and it is not permitted to short a security in which one has a long position to avoid realizing capital gains. Investors realizing such a "shorting-the-box-strategy" are treated as if they had sold the long position and hence their capital gains are taxed.

optimal to realize losses immediately in both tax-systems where capital losses can only be offset against capital gains and when the investor has only one risky asset in tax-systems with limited deduction of capital losses from other income. It further extends the approach of Ehling et al. (2008) by allowing for limited deductibility of capital losses from other income. In contrast to their setting and that of Constantinides (1983), here, it can be optimal to realize capital gains immediately, which significantly complicates the investment decision. Immediate realization of capital gains provides the investor with the opportunity of generating future capital losses that can be used to offset other income. The opportunity to offset income with realized capital losses thus provides the investor with the opportunity of sharing losses with the government.

Offsetting losses against other income is desirable for two reasons. First, it increases the investor's cash at hand that can be invested and earn profits immediately, while offsetting losses from realized capital gains only avoids tax-payments, but does not immediately increase the investor's wealth. Second, other income is usually subject to a higher tax rate than capital gains.

However, in a tax-system that allows for offsetting losses against other income, the investor has to decide whether to offset losses against realized capital gains or against other income. Since losses have to be offset against realized capital gains first, the decision to offset losses against other income requires the investor to limit the realization of capital gains and thus the decisions about how to use capital losses and how to allocate wealth to assets become interrelated.

The remainder of this paper proceeds as follows: section 2 presents our model and explains which factors driving portfolio choice are caused by limited capital loss deduction. Section 3 contains our numerical solution to the investor's life cycle consumption investment problem and section 4 concludes.

## 2 The Model

We consider the consumption-portfolio problem of a short-selling constrained investor in the presence of capital gains taxation and limited capital loss deductibility in discrete time. Our assumptions concerning the security market, the taxable treatment of profits, the optimal trading strategy in the presence of unrealized capital losses, and the investor's consumption portfolio problem are outlined below.

## 2.1 Investment Opportunity Set

The investment opportunity set our investor is facing consists of a risky dividend-paying stock and a risk-free money market account.[2] The money market account pays a post-tax return $r$. The pre-tax capital gains rate of the stock $g_t$ from time $t$ to $t+1$ is lognormally distributed with mean $\mu$ and standard deviation $\sigma$. Additionally, the stock pays a risk-free constant post-tax dividend rate $d$.

## 2.2 Taxable Treatment of Profits

We impose Constantinides (1983) assumptions (2) and (3); i.e., in our model, wash sales are permitted and long- and short-term capital gains are subject to the same tax rate. Income from interest and dividends is taxed at rate $\tau_i$.[3] Realized capital gains are taxable at rate $\tau_g \leq \tau_i$. The tax basis for equity currently held is the weighted average purchase price of the assets.

The focus of our analysis is a feature of the tax-code that – to the best of our knowledge – has not received any attention in the portfolio choice literature so far – the limited deductibility of capital losses against other income.

The common assumption in the portfolio choice literature dealing with capital gains taxes is that capital gains and losses are subject to the same taxable treatment (see e.g. Constantinides (1983), Dammon et al. (2001), DeMiguel and Uppal (2005), Gallmeyer et al. (2006), Garlappi et al. (2001), Huang (2008), Hur (2001)). We refer to such tax-systems as tax-systems with symmetrical treatment of capital gains and losses.

**Definition 2.1 (Symmetrical treatment, ST).** *A tax-system with symmetrical treatment of realized capital gains and losses is a tax-system in which the same tax rate applies to realized capital gains and capital losses. If the investor realizes a capital loss, there is an immediate tax rebate payment the investor can reinvest. We refer to this as a tax-system of the ST type.*

We consider the ST case as a benchmark in our analysis. The second tax-system we consider to be a benchmark is one in which realized capital losses can only be offset against realized capital gains, but not against other income. This type of tax-system is analyzed in Gallmeyer and Srivastava (2003) and Ehling et al. (2008). Such taxable treatment of capital gains can be found, for example, in the Canadian and several European tax codes.

[2] We focus on the one-asset case in this paper to keep our problem numerically tractable.

[3] Given the fact that the lower tax rate on dividend income is only applicable until 2010 and that from 2011 on it will rise to the tax rate on other income, we do not consider different tax rates on dividends and interest payments here.

**Definition 2.2 (No deductions, ND).** *In a tax-system with no deductions, the investor is compensated for realized capital losses with a tax loss carry-forward that can be used to offset realized capital gains. An amount that is not immediately used is carried over indefinitely and can be used to offset future capital gains. A tax loss carry-forward that has not been used by the end of an investor's life is forfeited. We refer to this as a tax-system of the ND type.*

In contrast to the ST case, the compensation for realized capital losses does not come as an immediate tax rebate payment, but as a tax loss carry-forward, which is a less attractive compensation for two reasons. First, in contrast to an immediate tax rebate payment, a tax loss carry-forward cannot earn any profits. Second, a tax loss carry-forward bears the risk of never being used and thus ending up worthless. This risk is especially relevant for older investors whose remaining investment horizon is shorter.

If capital losses are partly deductible from other income, a tax loss carry-forward might be a more attractive compensation than an immediate tax rebate payment at tax rate $\tau_g$ as in the ST case. This is due to the fact that the investor's tax rate on other income, $\tau_i$, usually exceeds the tax rate on capital gains, $\tau_g$, such that offsetting one dollar of tax loss carry-forward from other income decreases the investor's tax bill by a higher amount than offsetting that dollar against realized capital gains.

**Definition 2.3 (Limited deduction, LD).** *In a tax-system with limited deduction, an investor is compensated for realized capital losses with a tax loss carry-forward. This tax loss carry-forward has to first be used to offset realized capital gains. Each year, tax loss carry-forward not exceeding some finite amount $M$, can be deducted against other income. A tax loss carry-forward remaining after this procedure is carried over indefinitely. A tax loss carry-forward that has not been used by the end of an investor's life is forfeited. We refer to this as a tax-system of the LD type.*

If an investor in the LD case at time $t$ is endowed with an initial tax loss carry-forward $L_{t-1} \leq 0$ from the previous period, the tax loss carry-forward is used to offset realized capital gains. By $G_t$, we denote the investor's realized capital gain or loss at time $t$. The amount which is subject to the capital gains tax is given by

$$T_t \equiv \max\left(G_t + L_{t-1}, 0\right). \tag{1}$$

The tax loss carry-forward that remains after offsetting realized capital gains but before offsetting other income is given by

$$RL_t \equiv \min\left(G_t + L_{t-1}, 0\right). \tag{2}$$

If this remaining tax loss carry-forward $RL_t$ is negative, the lesser of the absolute value of the remaining tax loss carry-forward and some finite amount $M \geq 0$ can be used to offset other income. In the special case that $M = 0$, tax-systems of the LD type are concordant with those of the ND type. The amount deductible $D_t$ is thus given by

$$D_t \equiv \min\left(-RL_t, M\right). \tag{3}$$

That amount of the investor's remaining tax loss carry-forward that cannot be deducted from other income is carried over to the next period as a tax loss carry-forward $L_t$. It is given by

$$L_t \equiv RL_t + D_t. \tag{4}$$

There are two key differences between the tax-system of the LD type and those of the ST and the ND type.

First, in LD systems, the investor can use tax loss carry-forward in two different ways. In the ND case the investor can only use tax loss carry-forward against realized capital gains, so there is no incentive to defer the use of a tax loss carry-forward. In the ST case, the investor can never end up with a tax loss carry-forward. Only in the LD case can the investor make a decision on how the tax loss carry-forward shall be used, i.e., whether to use the tax loss carry-forward to offset realized capital gains or other income.

Second, in LD tax-systems, the investor has an incentive to realize more capital gains than required to rebalance her portfolio. This is due to the fact that applying capital losses to offset other income has two advantages compared to using them to offset realized capital gains. First, it increases the investor's total wealth, which allows earning more potential profit. Second, other income is usually subject to a higher tax rate than capital gains such that the tax advantage from using capital losses to offset other income outweighs the tax advantage of offsetting it against realized capital gains.[4]

In the ST and ND case, the only motive for selling equity with unrealized capital gains is

[4]The higher tax rate applicable to realized capital losses makes volatile assets appealing and can be a factor that helps explain the high price of some risky assets.

to rebalance the portfolio. In contrast, in the LD case, the investor has a second motive for realizing capital gains. By cutting capital gains short, that is realizing more capital gains than required to rebalance her portfolio, she creates the opportunity of using future capital losses to offset other income, which is usually subject to a higher tax rate than capital gains.[5] In other words, by cutting capital gains short, she pays $\tau_g$ dollars per unit of capital gains, but regains the option to utilize future losses against other income, subject to tax rate $\tau_i \geq \tau_g$. Therefore, in contrast to the ST and ND cases, besides making a decision on optimal consumption and the desired level of her equity exposure, an investor endowed with unrealized capital gains also has to make an informed decision on how much of her unrealized capital gains to cut short.

In summary, in the LD case, there are two reasons for realizing capital gains. First, the investor might want to rebalance her portfolio by selling equity. Second, the investor might want sell equity to regain the opportunity of creating future losses to use against other income by immediately repurchasing that equity. The first motive only affects the investor's equity exposure, but does not affect her unrealized capital gains per unit of equity. It also exists in tax-systems of the ST and ND type. The second motive does not affect her equity exposure, but only her unrealized capital gains per unit of equity. It only exists in tax-systems of the LD type.

## 2.3 Optimal Tax-Timing in the ND and the LD Case

Given assumptions (1) to (4), Constantinides (1983) shows that it is optimal to realize capital losses immediately. In fact, his proof also holds without imposing assumption (1), that investors do not face any short-selling constraints, and can also be applied for short-selling constrained investors. In this section, we argue that his proof can be generalized to tax-systems of the ND type and the one-asset case of tax-systems of the LD type by additionally dropping assumption (4), that there is no limit on tax rebates for incurred capital losses.

**Theorem 2.1.** *Consider a tax-system of the LD type when the investor holds only one risky asset or a tax-system of the ND type. When assumptions (2) and (3) hold, it is optimal to realize capital losses immediately, if* $\tau_i \geq \tau_g$.

A formal proof of theorem 2.1 is given in Appendix A. The economic intuition behind the theorem is as follows: since a tax loss carry-forward does not pay any interest, its value

[5] The rationale behind cutting gains short is similar to that in Constantinides (1984). While in his setting the reason is the different taxable treatment of long and short-term capital gains, in our setting the reason is the different tax rates applicable to capital gains and other income.

can never be above the maximum amount of wealth the tax loss can be converted into. This maximum amount is equal to the investor's tax rate on other income $\tau_i$ in the LD case and the capital gains rate $\tau_g$ in the ND case. The only way to receive compensation at tax rate $\tau_i$ is to generate a tax loss carry-forward, i.e. to realize the loss. Even if the investor cannot use her entire losses to offset other income immediately or trades in a tax-system of the ND type, it remains optimal to realize the entire loss due to the higher flexibility of the tax loss carry-forward compared with carrying unrealized capital losses that are tied to the asset and run the risk of getting lost in the case of a capital gain.

However, theorem 2.1 cannot be generalized to the multiple asset case in tax-systems of the LD type if $\tau_i \neq \tau_g$. In the multiple asset case with $\tau_i > \tau_g$ the investor can end up with one asset encountering unrealized capital gains and one asset being facing unrealized capital losses. When the investor wants to realize some of the capital gains to rebalance her portfolio, it might be optimal to postpone the realization of the capital losses to avoid using them against the capital gains in the present period and retain the opportunity of using them against other income in some forthcoming period. Since losses have to first be used against realized capital gains, unrealized capital losses give rise to a timing option – the investor can decide when to realize them. By choosing periods in which no capital gains are realized, the investor can use her losses against other income.

In the multiple asset case with $\tau_i < \tau_g$, using losses against other income that is subject to a lower tax rate is less desirable than offsetting losses against realized capital gains. Consequently, it can be optimal not to realize all losses in the absence of capital gains for that period to avoid applying them to tax rate $\tau_i$. However, in most tax-systems found around the world, the tax rate on other income is usually not below the tax rate on capital gains such that this second case is less important in practice.

## 2.4 The Life Cycle Model

We consider an economy consisting of short-selling constrained investors living for at most $T$ years, who can only trade at time $t = 0, 1, \ldots, T$. The investor derives utility from consumption and bequest. The investor's utility function is of the CRRA-type with parameter of risk-aversion $\gamma \in [0, \infty)$. The parameter $\gamma$ represents the investor's willingness to substitute consumption among different points in time. It also represents the elasticity of consumption, which is given by $\frac{1}{\gamma}$. For simplicity, we assume that all income is derived from financial assets. We thereby implicitly assume that losses even qualify for reductions in the investor's tax bill and tax rebate payments if the investor is not endowed with other income. Losses not exceeding a constant amount of $M$ qualify for tax rebate payments and are subject to tax rate $\tau_i$.

By $\theta_t$ we denote the fraction of the investor's capital gains that are realized to cut capital gains short without changing the investor's equity exposure, i.e. by immediately repurchasing the equity sold. By $P_t$ we denote the price of the stock at the beginning of period $t$. By $P_t^*$ we denote the investor's average historical purchase price after trading at time $t$. Even though under current tax-law, investors can choose the exact basis rule, we follow recent portfolio choice literature including Dammon et al. (2001) and Ehling et al. (2008) to make our results comparable to theirs and keep our optimization problem numerically tractable. The impact of the basis rule is discussed in more detail in section 3.5.3. $q_t$ denotes the number of stocks the investor holds from time $t$ to $t+1$. The total number $N_t$ of units of the stocks that are sold at time $t$ is then given by

$$N_t = \max\left(q_{t-1} - q_t, 0\right) + \min\left(q_{t-1}, q_t\right)\theta_t. \tag{5}$$

The first summand in equation (5) defines the number of units of stocks sold to reduce the investor's equity exposure after trading. It does not affect the amount of unrealized gains per stock. The second summand denotes the number of stocks sold and immediately repurchased to cut gains short. It affects the amount of unrealized gains per stock, but leaves the investor's equity exposure unaffected.

If the investor faces unrealized capital losses, in our model with one risky asset, it is optimal to realize these losses immediately and repurchase to retain the desired equity exposure. Consequently, the purchase price after trading is equal to the current market price, i.e. $P_t^* = P_t$ if $P_{t-1}^* \geq P_t$.

If, on the other hand, the investor faces unrealized capital gains, her purchase price $P_t^*$ is a weighted average of her historical purchase price and the current market price. The weight assigned to the historical purchase price is given by the number of stocks the investor holds after realization of capital gains $q_{t-1} - N_t$. The weight assigned to the current market price is given by the number of stocks the investor purchases at time $t$. It is given by the sum of the number of stocks $\max\left(q_t - q_{t-1}, 0\right)$ the investor purchases to increase her equity exposure and the number of stocks $\min\left(q_t, q_{t-1}\right)\theta_t$ the investor repurchases immediately after having sold them to cut unrealized capital gains short. Consequently,

$$P_t^* = \begin{cases} \frac{[q_{t-1}-N_t]P_{t-1}^* + [\max(q_t - q_{t-1},0)+\min(q_{t-1},q_t)\theta_t]P_t}{q_t} & \text{if } P_{t-1}^* < P_t \\ P_t & \text{if } P_{t-1}^* \geq P_t. \end{cases} \tag{6}$$

The investor's realized capital gains or losses, $G_t$, in period $t$ are given by

$$G_t = \left[\chi_{\{P_t > P_{t-1}^*\}} N_t + \chi_{\{P_t \leq P_{t-1}^*\}} q_{t-1}\right] \left(P_t - P_{t-1}^*\right) \tag{7}$$

where $\chi_{\{P_{t-1}^* > P_t\}}$ denotes the characteristic function, which is one for $P_{t-1}^* > P_t$ and zero otherwise.

By $R \equiv 1 + r$ we denote the gross after-tax return on the risk-free asset. $b_t$ is the number of units of the risk-free asset with the purchase price of one that the investor holds from time $t$ to $t+1$. $W_t$ is the investor's beginning-of-period $t$ wealth before trading, $C_t$ is the investor's period $t$ consumption. $i$ is a constant inflation rate. It is assumed that bequeathed wealth is used to purchase an $H$-period annuity and that this $H$-period annuity provides the beneficiary with nominal consumption of $A_H W_{t+1} (1+i)^{k-t}$ at date $k$ $(t+1 \leq k \leq t+H)$, in which $A_H \equiv \frac{r^*(1+r^*)^{H-1}}{(1+r^*)^H - 1}$ is the $H$-period annuity factor and $r^*$ is the after-tax real bond return. $F(t)$ denotes the time 0 probability that the investor is still alive through period $t$ $(t \leq T)$. The parameter $\beta$ represents the investor's annual utility discount factor.

The investor's optimization problem is then given by

$$\max_{C_t, q_t, \theta_t} \mathbb{E}\left[\sum_{t=0}^{T} \beta^t \left(F(t-1) U\left(\frac{C_t}{(1+i)^t}\right) + (F(t-1) - F(t)) \sum_{k=t+1}^{t+H} \beta^{k-t} U\left(\frac{A_H W_{t+1}}{(1+i)^{t+1}}\right)\right)\right] \tag{8}$$

s.t.

$$W_t = q_{t-1}(1+d) P_t + b_{t-1}(1+r), \qquad t = 0, \ldots, T+1 \tag{9}$$

$$W_t = \tau_g T_t + q_t P_t + b_t + C_t - \tau_i D_t \qquad t = 0, \ldots, T \tag{10}$$

$$q_t \geq 0, b_t \geq 0 \qquad t = 0, \ldots, T \tag{11}$$

and equations (1) to (4) given the initial holding of bonds $b_{-1}$, stocks $q_{-1}$, the initial tax-basis $P_{-1}^*$, the price of one unit of the stock $P_0$, the initial wealth $W_0$ and the initial tax loss carry-forward $L_{-1}$.

According to equation (8), the investor maximizes discounted expected utility of lifetime consumption and bequest. Equation (9) defines the investor's beginning-of-period $t$ wealth as the sum of wealth in stocks and bonds before trading at time $t$, including after-tax interest and dividend income, but before any capital gains taxes resulting from trading at time $t$. Equation (10) is the investor's budget constraint at time $t$. If the investor trades equity, $T_t$ is subject to the capital gains tax rate $\tau_g$ and $D_t$ qualifies for tax rebate payments subject to tax rate $\tau_i$.

By $X_t$, we denote the vector of the investor's state variables, $V_t(.)$ the investor's value function at time $t$, and $f(t)$ the probability of surviving from period $t$ to $t+1$ given the investor is alive at the beginning of period $t$. If we take into account that the sum in the last term of the objective function (8) can be simplified by making use of the fact that $\sum_{k=t+1}^{t+H} \beta^{k-t} = \frac{\beta(1-\beta^H)}{1-\beta}$, we can rewrite the Bellmann equation for the optimization problem as

$$\begin{aligned} V_t(X_t) \quad = \quad \max_{C_t, q_t, \theta_t} & \left[ f(t) U\left(\frac{C_t}{(1+i)^t}\right) + f(t)\beta \mathbb{E}_t\left[V_{t+1}\left(X_{t+1}\right)\right] \right. \\ & \left. + (1-f(t)) \frac{\beta\left(1-\beta^H\right)}{1-\beta} U\left(\frac{A_H W_{t+1}}{(1+i)^{t+1}}\right) \right] \end{aligned} \tag{12}$$

for $t = 0, \ldots, T-1$ subject to equations (1), (4), (6), (7), and (9) to (11) with terminal condition

$$V_T\left(X_T\right) = \max_{C_T, q_T, \theta_T} U\left(\frac{C_T}{(1+i)^T}\right) + \frac{\beta\left(1-\beta^H\right)}{1-\beta} \mathbb{E}\left[U\left(\frac{A_H W_{T+1}}{(1+i)^{T+1}}\right)\right]. \tag{13}$$

The state variables required to solve the problem at time $t$ are the investor's beginning-of-period wealth $W_t$ before trading, the initial tax loss carry-forward $L_{t-1}$, the price of the stock $P_t$, its tax basis $P^*_{t-1}$, and the number of stocks $q_{t-1}$ the investor holds at the beginning of period $t$ before trading. Thus, the vector of state variables $X_t$ at time $t$ can be represented as

$$X_t = [P_t, W_t, L_{t-1}, P^*_{t-1}, q_{t-1}]. \tag{14}$$

We rewrite the optimization problem by normalizing with the investor's beginning-of-period wealth $W_t$, and use the relation between $P^*_{t-1}$ and $P_t$ as a state variable, which allows us to reduce the number of state variables to four: the investor's basis-price-ratio $p^*_{t-1} \equiv \frac{P^*_{t-1}}{P_t}$, her initial equity exposure $s_t \equiv \frac{q_{t-1}P_t}{W_t}$, her initial tax loss carry-forward to wealth ratio $l_{t-1} \equiv \frac{L_{t-1}}{W_t}$ and the fraction of total wealth qualifying for tax rebate payments $m_t \equiv \frac{M}{W_t}$. We solve the rewritten optimization problem by backward-induction. The technical details can be found in Appendix B.

## 2.5 Base Case Parameter Values

For the numerical analysis, it is assumed that annual inflation is $i = 3.5\%$. The tax rate on realized capital gains is assumed to be $\tau_g = 20\%$. The tax rate on interest and dividends is assumed to be $\tau_i = 35\%$. In line with current tax-law, we assume that the maximum amount of losses qualifying for tax rebate payments subject to tax rate $\tau_i$ is $M = 3,000$.

The pre-tax risk-free rate is 6% such that the after-tax risk-free rate is $r = 3.9\%$. The return on equity consists of both capital gains and dividend payments. Capital gains are lognormally distributed, serially independent, come with an expected return of $\mu = 7\%$ and a standard deviation of $\sigma = 20.7\%$. The pre-tax dividend payments are 2% of the stock's value in each period such that the after-tax dividend rate is $d = 1.3\%$. The correct choice of equity premium has been subject to much theoretical and empirical research (see Siegel (2005) for a survey). While the historical risk-premium has been about 6% (Mehra and Prescott (1985)) in the US since 1872, economists doubt whether this will be true in future periods. We follow the current consensus, which is about 3% to 4% (see e.g. Cocco et al. (2005), Dammon et al. (2001), Ehling et al. (2008), Fama and French (2002), Gallmeyer et al. (2006) and Gomes and Michaelides (2005)).

We assume the investor makes decisions annually starting at age 20 ($t = 0$). The maximum age the investor can attain is set to 100 years ($T = 80$). It is further assumed that the relative risk-aversion of the investor is $\gamma = 3$ and the annual utility discount factor is $\beta = 0.96$. $H$ is set to $H = 60$ in the bequest function, indicating that the investor wishes to provide the beneficiary with an income stream for the next 60 years. The data for the survival probabilities of our investor are taken from the 2001 Commissioners Standard Ordinary Mortality Table for female investors. Table 1 summarizes our choice for the base-case parameter values.

Table 1 about here

# 3 Numerical Evidence

Having introduced the taxable treatment of realized capital gains and losses in three different types of tax-systems and also the investor's optimization problem, we now turn to its numerical solution. We first analyze our base-case scenario and contrast the optimal conditional investment strategies for the three different types of tax-systems in section 3.1. Section 3.2 analyzes when it is optimal to cut gains short. The impact of an initial tax loss carry-forward on optimal investment strategies is discussed in section 3.3. In section 3.4, we quantify the effective tax rate that would make an investor indifferent between being compensated for a tax loss carry-forward immediately and keeping the tax loss carry-forward to use it in forthcoming periods. Section 3.5 summarizes the results of a Monte Carlo analysis on the evolution of an investor's optimal consumption investment strategy over her life cycle. Section 3.6 discusses the impact of tax rebate payments on the investor's welfare.

## 3.1 Optimal Investment Policy without initial Tax Loss Carry-Forward

We begin the discussion of our numerical results by first considering the optimal investment policy of an investor who is not endowed with an initial tax loss carry-forward. In general, the investor's optimal equity exposure depends on her basis-price-ratio, her initial equity exposure, her initial tax loss carry-forward and her level of wealth. Her basis-price-ratio indicates whether the investor faces an unrealized capital gain (basis-price-ratio below one) or loss (basis-price-ratio above one). The basis-price-ratio thereby indicates potential tax payments or tax loss carry-forwards granted when selling equity. The investor's initial equity exposure indicates to which extent the investor is affected by unrealized capital gains or losses per unit of equity. An initial tax loss carry-forward provides the investor with the opportunity of either avoiding capital gains tax payments by using her tax loss carry-forward against realized capital gains or offseting other income. The investor's wealth level affects the investor's optimal investment decision as it determines which maximum fraction of losses from the total wealth can be offset against other income. Since $M$ is a constant amount, the fraction of losses than can be offset by applying them to other income is higher for investors with low wealth levels than for investors with high wealth levels. The length of the remaining investment horizon also has an impact on the investor's optimal equity exposure due to the fact that a tax loss carry-forward cannot be bequeathed and unrealized capital gains are forgiven at death and thereby escape taxation.

Figure 1 about here

Figure 1 depicts the relation between the optimal equity exposure of an investor at age 30 who is not endowed with an initial tax loss carry-forward and the investor's initial basis-price-ratio as well as her initial equity exposure. The upper graphs depict the investor's optimal equity exposure in ST tax-systems (upper left graph) and the ND type (upper right graph), which have been dealt with in the work of Dammon et al. (2001) and Ehling et al. (2008). The lower graphs show her optimal equity exposure in a tax-system of the LD type when being endowed with an initial level of wealth before trading of $ 3,000 (lower left graph), and $ 3,000,000 (lower right graph), respectively.

The optimal investment policies in LD tax-systems depends heavily on the investor's wealth level. An investor with an initial wealth level of $ 3,000 (lower left graph) increases her equity exposure monotonically as her basis-price-ratio rises. When the investor is endowed with an initial basis-price-ratio above one, indicating that the investor faces unrealized capital losses, she optimally realizes these losses immediately. This leaves the investor with an immediate tax rebate payment for all incurred capital losses and increases her wealth level. The more the level of unrealized losses per unit of equity increases and the higher the investor's initial equity exposure, the more her overall wealth level increases. The higher the unrealized capital losses per unit of equity, i.e. the higher the investor's basis-price-ratio, and the higher the investor's initial equity exposure is, then the more her wealth level increases. As we defined the optimal equity exposure as the fraction of the investor's equity after trading relative to her beginning-of-period wealth, the optimal equity exposure increases when the investor's wealth level after trading increases, which is the case, for example, when she receives tax rebate payments.

When the investor faces unrealized capital gains, she has to decide whether or not to cut these gains short to regain the opportunity of offsetting potential future capital losses against other income. Cutting gains short is more desirable the higher the investor's potential future tax rebate payments are, relative to total wealth. For investors with low levels of wealth, the fraction of capital losses that can be used against future income is substantial. Consequently, an investor with a low wealth level is more likely to realize her capital gains. Due to the tax payments associated with the cutting of her unrealized gains, her wealth level decreases, which is why the investor's optimal equity exposure decreases as her initial equity exposure increases and her basis-price-ratio decreases.

For an investor who is endowed with an initial wealth level of $ 3,000,000 (lower right graph), the optimal equity exposure is substantially lower. Additionally, the impact of her basis-price-ratio and initial equity exposure on her optimal equity exposure differs fundamentally from that of the investor with $ 3,000 initial wealth. The difference in the optimal equity exposure between the two graphs arises from the different fraction of potential losses that can be used against other income. The investor being endowed with a low wealth level of only $ 3,000 can offset all potential losses against other income. This is not true for the investor who is endowed with an initial wealth level of $ 3,000,000. Such an investor can only offset a very small fraction of her capital gains against other income such that her investment decision becomes quite similar to that of an investor in a tax-system of the ND type (upper right graph) who cannot offset any capital losses from other income. The optimal equity exposure of investors with either unrealized capital gains or losses exceeds that of an investor who is endowed with neither unrealized capital gains nor losses.

The reasons behind the higher equity exposure for investors with unrealized capital gains versus unrealized losses, however, are remarkably different. Being endowed with unrealized capital gains, the investor seeks to avoid capital gains tax payments and therefore accepts higher equity exposure. Especially if equity has performed well in the past, its fraction relative to the investor's total wealth has been increasing, which might have resulted in an unbalanced portfolio. To avoid the capital gains tax payment, however, the investor might accept a deviation from her otherwise desired equity exposure – such investors are also referred to as being locked in to their capital gains. This deviation is higher when her basis-price-ratio is lower, i.e. when her unrealized capital gains per unit of equity are higher and would thereby invoke higher tax costs for rebalancing her portfolio.

In contrast, an investor endowed with an unrealized capital loss, optimally realizes that loss immediately, leaving her with a tax loss carry-forward. In tax-systems of the ND type and tax-systems of the LD type where the investor is endowed with substantial wealth and can only offset proportionally small amounts of losses against other income, this tax loss carry-forward allows the investor to earn some future capital gains tax-free. Hence, the after-tax risk-return profile of equity becomes more desirable. Consequently, the resulting optimal equity exposure is above the level of an investor who is not endowed with an unrealized capital loss.

The results in the upper graphs confirm the results of recent literature on optimal investment decisions in tax-systems of the ST and the ND type (see Dammon et al. (2001) and Ehling et al. (2008)). Since tax-systems of the ST type (upper left graph) provide the investor with more generous compensation for realized capital losses, it is not surprising that optimal equity exposure in such tax-systems is above the optimal equity exposure in tax-systems of the ND type (upper right graph).

The taxable treatment of capital losses in LD tax-systems is more attractive for an investor than in tax-systems of the ND type due to the opportunity to use losses against other income. While this causes investors to increase their equity exposure when they have low wealth levels such that they can use a substantial fraction of potential losses against other income, this advantage becomes negligible to investors that are endowed with substantial wealth and can only use small amounts of potential losses against other income.

In sum, while in tax-systems of the ST and the ND type, the homogeneity of the CRRA utility function assures that the investor's wealth level does not have an impact on her investment decision, this is not true in tax-systems of the LD type, where the wealth level affects the fraction of losses that can be used against other income. Since the tax rate $\tau_i$, applicable to he loss being used against other income, exceeds the tax rate on capital gains $\tau_g$, it can be optimal to cut capital gains short to regain the opportunity of offsetting losses at tax rate $\tau_i$.

## 3.2 When to Cut Gains Short

Analyzing the differences between the optimal equity exposure for an investor with low and high wealth levels in a tax-system of the LD type, we argued that it might be optimal to cut capital gains short to regain the opportunity of using potential future losses against other income. Furthermore, our results in section 3.1 indicate that the investor's optimal equity exposure depends crucially on her wealth level.

Figure 2 about here

Figure 2 depicts the relation between the initial wealth level and optimal fraction of capital gains to cut short for a 30 year old investor who is not endowed with a tax loss carry-forward and whose initial equity exposure is 60%. If the investor faces unrealized capital gains, her optimal equity exposure depends on whether she cuts these gains short or not.

If she does not cut her gains short, each trade has an impact on her basis-price-ratio if she buys equity or her tax payments if she sells equity. If, however, the investor cuts all her capital gains short, she can then choose her desired equity exposure without facing any additional tax consequences or changes in her basis-price-ratio.

Figure 2 shows that the investor optimally realizes all capital gains when her wealth level is small. When her wealth level is substantial, however, she does not cut any capital gains short. Her only motive for realizing capital gains is rebalancing her portfolio. This dependency of the optimal realization of capital gains and the investor's wealth level is again due to the fact that the investor can only offset a constant amount of capital losses each year. Consequently, if the investor's wealth level is small, she can use a substantial fraction of losses against other income. If, however, her initial wealth level is substantial, the fraction is small.

The reason for cutting gains short is the advantage that comes from using capital losses against other income. Since the advantage the investor yields from cutting gains short is substantial when her wealth level is small and small when her wealth level is large, she optimally cuts gains short when her wealth level is small and does not cut her gains short when her wealth level is substantial. The cut-off point is at around $ 400,000, such that investors with less than $ 400,000 tend to cut their gains short and investors with higher wealth levels tend not to cut their gains short.

## 3.3 Investment Policy with initial Tax Loss Carry-Forward

So far, we have considered the optimal investment strategy of an investor who is not endowed with an initial tax loss carry-forward. An investor who is endowed with an initial tax loss carry-forward has to make an informed decision on whether to realize capital gains and use the tax loss carry-forward against these gains or to postpone the realization of capital gains and to use the tax loss carry-forward against other income.

Figure 3 about here

Figure 3 depicts the optimal equity exposure (left graph) and optimal fraction of gains to cut short (right graph) for an investor at age 30 who is endowed with an initial wealth level of $ 3,000 and a tax loss carry-forward of -30% of her initial wealth (i.e. a tax loss carry-forward of $ -900).[6]

If the investor faces substantial unrealized capital gains, which is the case if the investor's basis-price-ratio is small and her initial equity exposure is high, the investor optimally realizes her capital gains immediately and uses her tax loss carry-forward to offset these realized capital gains. Even though her tax rate on capital gains is substantially below her tax rate on other income, which she could earn by postponing the realization of capital gains by one period, she cuts her capital gains short to rebalance her portfolio.

In total, cutting capital gains short has three consequences. First, the investor can reduce the initial equity exposure to her desired equity exposure. Second, the investor can offset future capital losses against other income. Third, the investor has to use her present tax loss carry-forward against her realized capital gains first.

While the third factor suggests that the investor should postpone the realization of her

[6]We are aware of the unlikelihood of an investor being endowed with such a large tax loss carry-forward when her wealth level is so small. However, the case analyzed in this section facilitate understanding of the effects driving optimal investment strategies in tax-systems of the LD type.

capital gains, the first two factors suggest that the investor should cut her capital gains short immediately. The first factor is crucial if the investor's initial equity exposure deviates substantially from her desired equity exposure. The second factor becomes more important the higher the unrealized capital gains per unit of equity are. For investors with low unrealized capital gains, the advantage from cutting unrealized capital gains short immediately is small because there is a higher likelihood of these minimal gains turning into losses in the future than with higher levels of capital gains. Thus, the investor is more likely to use her present tax loss carry-forward against other income. As a result, the investor's optimal equity exposure is higher with small unrealized capital gains than with substantial unrealized capital gains.

### 3.4 Effective Tax Rate on Tax Loss Carry-Forward

In this section, we analyze the effective tax rate $\tau_e$ applicable to the investor's tax loss carry-forward that would make the investor indifferent between the options of immediately receiving a tax rebate payment and keeping the tax loss carry-forward to offset other income or realized capital gains in forthcoming periods.

Since in tax-systems of the LD type each dollar of tax loss carry-forward allows the investor to decrease tax-payments by not more than $\tau_i$ dollars, one unit of tax loss carry-forward cannot be worth more than these $\tau_i$ dollars. However, if the investor is endowed with a high level of wealth and faces a significant tax loss carry-forward, her effective tax rate might be worth less than $\tau_i$ dollars for three reasons. First, she might not make use of her entire tax loss carry-forward in her life, implying that the potential value of the tax loss carry-forward would never turn into wealth that could be consumed or bequeathed. This type of risk is most important for older investors facing higher mortality rates. Second, even if the investor can make use of her entire tax loss carry-forward, it might take several periods until her entire tax loss carry-forward is converted to wealth and she can earn profits from it. Third, she might want to offset parts of her tax loss carry-forward against realized capital gains, which are subject to $\tau_g \leq \tau_i$. Consequently $\tau_e \leq \tau_i$.

In tax-systems of the ND type, each dollar of tax loss carry-forward cannot be worth more than $\tau_g$ dollars since the investor can only offset losses against realized capital gains which are subject to a tax rate of $\tau_g$. Since the tax loss carry-forward does not pay any interest, whereas tax rebate payments can be reinvested and do yield profits, in ND tax-systems, one unit of a large tax loss carry-forward should be worth less than one unit of a small tax loss carry-forward. As a result, the effective tax rate should decrease as the level of the investor's tax loss carry-forward increases.

This relation does not hold true in tax-systems of the LD type. In these tax-systems, the value of the tax loss carry-forward depends on whether it is used against other income or against realized capital gains.

Figure 4 about here

Figure 4 depicts the relation between the investor's effective tax rate and her initial equity exposure as well as the level of her initial tax loss carry-forward to wealth ratio. The left graph shows the impact of the investor's initial equity exposure and the level of her tax loss carry-forward for an investor at age 30 in a tax-system of the ND type, the right graph for an investor in a tax-system of the LD type who is endowed with an initial wealth level of $ 3,000.

The left graph shows that for investors facing unrealized capital gains in tax-systems of the ND type, the effective tax rate increases as the initial equity exposure does, and decreases as the level of the tax loss carry-forward increases (in absolute value). The first scenario results from the fact that an investor endowed with unrealized capital gains and substantial initial equity exposure, tends to make use of her tax loss carry-forward earlier than an investor being endowed with a small initial equity exposure. Consequently, the average waiting time until the tax loss carry-forward is used and provides the investor with the opportunity of earning profits is shorter, which is why the effective tax increases as the investor's initial equity exposure does.

The effective tax rate decreases as the level of tax loss carry-forward increases since a high level of tax loss carry-forward carries a higher probability of forfeiting some part of the loss. Even if it is entirely used, the average waiting time before its usage is longer. Consequently, the compensation the investor asks for per dollar of tax loss carry-forward to make her indifferent between receiving that compensation immediately and keeping her tax loss carry-forward for future periods is less as her level of tax loss carry-forward increases.

If the investor's initial equity exposure is substantial and her tax loss carry-forward is small, the effective tax rate reaches its maximum value of $\tau_g$, indicating that the investor would make use of her entire tax loss carry-forward immediately to reduce her equity exposure and rebalance her portfolio. She would thus be completely indifferent between receiving a tax rebate of $\tau_g$ dollars and using her tax loss carry-forward directly.

For an investor in a tax-system of the LD type being endowed with an initial wealth level of $ 3,000 (right graph), the relation between the investor's effective tax rate and her initial equity exposure and initial tax loss carry-forward looks entirely different. In contrast to the situation in ND tax-systems, here, the effective tax rate decreases as the investor's initial equity exposure increases and it increases as the investor's initial tax loss carry-forward increases (in absolute value). In contrast to tax-systems of the ND type, in tax-systems of the LD type the investor can use her tax loss carry-forward in two different ways. The investor can use the tax loss carry-forward against either realized capital gains or other income. The effective tax rate depends heavily on how the investor uses her tax loss carry-forward.

For high levels of initial equity exposure, the diversification motive and the desire not to get locked in outweighs the incentive to postpone the realization of capital gains to offset losses against other income. Hence, with low levels of initial tax loss carry-forward and high levels of initial equity exposure, the investor tends to cut her capital gains short, which forces her to use her tax loss carry-forward against these realized capital gains such that the effective tax rate is equal to the tax rate on capital gains. As the level of the investor's initial tax loss carry-forward increases in absolute value, there eventually comes the point where it is no longer optimal to cut capital gains short.

Since the tax rate on other income is higher than the tax rate on capital gains, this causes the effective tax rate to increase substantially. However, since the investor still wants to rebalance her portfolio, the investor has to use some part of her tax loss carry-forward against capital gains. The fraction of her tax loss carry-forward that offsets capital gains increases as her initial equity exposure increases, which is why her effective tax rate decreases as her initial equity exposure increases.

### 3.5 Unconditional Strategies

Having analyzed the investor's optimal investment policy given specific values of the state variables, we next turn to the investor's optimal unconditional investment policy over the life cycle. While the graphs in figures 1 to 3 provide a good impression about the impact of the state variables on the investor's optimal equity exposure and the tax-effects that drive these results, they do not reveal how likely the investor is to end up in which state. An investor who cuts capital gains short each period is, for example, very unlikely to end up with substantial unrealized capital gains.

To analyze the investor's optimal investment strategy over the life cycle, we run 50,000 simulations on our optimal grids in tax-systems of the LD, the ND and the ST types. We consider an investor who enters the market at age 20, who faces neither unrealized capital gains nor losses, who is not endowed with an initial tax loss carry-forward and whose initial wealth is \$ 10,000. In the LD case we additionally run a simulation with an initial wealth level of \$ 100,000 to explore the impact of the wealth level on optimal life cycle investment strategies.

In tax-systems with a tax-timing option, there are two reasons why an investor might choose a high equity exposure. First, equity has an appealing risk-return profile. In tax-systems of the LD type, the after-tax risk-return profile of the risky asset depends on the investor's wealth level, which determines the fraction of losses that can be offset against other income. Besides human capital and the flexibility of labor supply (Bodie et al. (1992)), information costs (Haliassos and Bertaut (1995)), changing risk aversion with age (Ballente and Green (2004)) and cointegration of stock and labor markets (Benzoni et al. (2007)), the lower fraction of losses that can be offset against other income with increasing wealth level is another reason why private investors might want to decrease their equity exposure over the life cycle.

Second, the investor might be locked in to her capital gains and wants to avoid the tax payments she is confronted with when selling equity. Especially when the investor is older and faces higher mortality rates, this motive is very important since the step up of the tax-basis for assets bequeathed allows the investor to entirely escape the taxation of her capital gains.

In tax-systems of the LD type, there is even a third reason for holding a higher equity exposure: the investor might want to offset a tax loss carry-forward against other income and therefore postpone the realization of capital gains.

We first present the results of our simulations in the base-case parameter setting in subsection 3.5.1. In subsection 3.5.2 we analyze the impact of the investor's tax rate on individual income on optimal portfolio choice. In subsection 3.5.3 we discuss the impact of the basis-rule on our results. In subsection 3.5.4 we sketch the impact of non-financial income on the incentive to cut capital gains short.

### 3.5.1 Base-Case Parameter Setting

We first consider the investor's optimal life cycle investment strategy in our base-case parameter setting.

Table 2 about here

In table 2, we summarize the evolution of the investor's basis-price-ratio and her optimal investment decisions over the life cycle from 50,000 simulations on the optimal path for an investor trading in ST, ND and LD tax-systems. We used the same realizations of stochastic capital gains on the risky asset in all simulations to make sure that our results are comparable with one another. Panel A depicts our results for an investor at age 30, panel B for an investor at age 60, and panel C for an investor at age 90. We show the mean, standard deviation, and percentiles of the distribution of the investor's optimal equity exposure $\alpha_t$ and her basis-price-ratio before trading $p^*_{t-1}$ for tax-systems of all three types. We further show the optimal cutting of capital gains $\theta_t$ and the fraction of losses $F_\Sigma^{(I)}$ the investor has offset so far against other income over the life cycle for the tax-system of the LD type.

As argued above, the investor's wealth level does not have an impact on her optimal investment decision in tax-systems of the ST and the ND type due to the homogeneity of the CRRA utility function. However, it significantly affects optimal investment decisions in tax-systems of the LD type. The columns marked $\text{LD}^4$ refer to an investor whose initial wealth level at age 20 is $\$\,10,000 = 10^4$, and columns marked $\text{LD}^5$ refer to an investor whose initial wealth level at age 20 is $\$\,100,000 = 10^5$.

Table 2 confirms that an investor trading in an LD tax-system holds substantially more equity when endowed with a low initial wealth level of \$ 10,000 than with a higher initial wealth level of \$ 100,000. The after-tax risk-return profile of the risky asset is more appealing for the former type of investor since a higher fraction of potential losses can be used against other income thus qualifying the investor for substantial tax rebate payments.

Since the opportunity of using potential capital losses against other income is very appealing, the investor cuts all her unrealized capital gains short when young and most when middle-aged. When endowed with a high wealth level, the investor decreases the fraction of capital gains being cut short. Consequently, the $\text{LD}^5$ investor faces higher unrealized capital gains, which can be seen from the evolution of the investor's basis-price-ratio. Hence, she tends to become locked in earlier than the $\text{LD}^4$ investor.

The average percentage of losses $F_\Sigma^{(I)}$ that has been used against other income at tax rate $\tau_i$ at age 30 is 100% in the $\text{LD}^4$ case, but only 79% in the $\text{LD}^5$ case. At age 60 this percentage declines to 98% and 73%, respectively. At age 90, the average percentage of losses the investor has used against other income is 95% in the $\text{LD}^4$ case and 74% in the $\text{LD}^5$ case. These values indicate that investors in the $\text{LD}^5$ case are more likely to use a tax loss carry-forward to rebalance their portfolios.

At the age of 90, neither $\text{LD}^4$ nor $\text{LD}^5$ investors cut their capital gains short. This result is caused by the reset provision of the tax code according to which unrealized capital gains are forgiven upon bequeath. Hence, in line with the findings of Dammon et al. (2001) and Ehling et al. (2008), the high level of the older investor's equity exposure is driven by higher mortality rates and the desire to postpone the realization of capital gains to fully escape the capital gains tax.

Investing in a tax-system of the LD type is ceteris paribus more attractive than investing in a tax-system of the ND type due to the opportunity of getting tax rebate payments for realized capital losses. Consequently, at young age, the investor's equity exposure in the LD tax-system is higher than in the ND tax-system. The difference in the investor's equity exposure is driven by the fraction of losses qualifying for tax rebate payments. Since rich investors can use only small fractions against other income, their equity exposure should be optimally smaller. Since the investor never cuts unrealized capital gains short in tax-systems of the ND type, she tends to become locked in significantly earlier, which can be seen by comparing the evolution of the investor's basis-price-ratios over the life cycle. Consequently, when the investor gets older, her equity exposure in tax-systems of the ND type increases faster than in tax-systems of the LD type, where diversification can be achieved with lower tax payments.

The desirability of investing in an LD versus ST tax-system depends crucially on the investor's wealth level. For an investor with a very small wealth level, investing in an LD tax-system is more desirable due to the higher tax rebate payments on realized capital losses. For an investor with a very high wealth level, however, investing in tax-systems of the ST type is more desirable, since such a tax-system does not limit the amount of losses that can be offset against other income. In the cases $LD^4$ and $LD^5$ analyzed here, the advantage of the higher tax rebate payments for realized capital losses in tax-systems of the LD type outweighs the advantage of unlimited tax rebate payments in tax-systems of the ST type for young investors as opposed to with extremely wealthy investors.

While an $LD^4$ investor still chooses a higher equity exposure at the age of 60, and $LD^5$ investor does not. In the course of time, the investor's wealth level increases and consequently, the fraction of losses that qualifies for tax rebate payments decreases. As a result, the after-tax risk-return-profile of the risky asset becomes less desirable.

Investing in a tax-system of the ST type is ceteris paribus more desirable than investing in a tax-system of the ND type due to the tax rebate payments for realized capital losses. While at young age, this causes the investor to choose a slightly higher equity exposure, we confirm the finding of Ehling et al. (2008) that the differences in the investor's investment strategies and her basis-price-ratio become negligible once the investor is locked in to her capital gains for both tax-systems. As soon as the investor is locked in, she has a strong incentive not to realize her capital gains to prevent having to make tax payments. Consequently, the investment decisions that investors in ST and ND tax-systems face once they are locked in are very similar, which is why the evolution of their investment strategies and state variables does not differ substantially.

The key difference between tax-systems of the LD type on the one hand and tax-systems of the ND and ST types on the other hand is the fact that cutting unrealized capital gains short is not desirable in tax-systems of the ND and the ST type, but can be optimal in tax-systems of the LD type to regain the opportunity of using capital losses against other income – thereby qualifying for higher tax rebate payments. In tax-systems of the ND and ST type there is no such incentive to cut realized capital gains short, which is why investors in these tax-systems tend to become locked in to their capital gains early. The $LD^5$ case shows that even with a substantial initial wealth level there is an incentive to cut gains short. In the $LD^4$ case, the investor even tends to realize all capital gains when young such that the distribution of her initial basis-price ratio at age 30 and age 60 does not differ substantially. Consequently, the opportunity to offset losses against other income provides a strong incentive to cut capital gains short which leaves private investors with well-diversified portfolios. In contrast, investors in tax-systems of the ND or ST type, since not facing an incentive to cut capital gains short, tend to become locked in to their capital gains, which leaves them with unbalanced portfolios. Our results suggest that the opportunity of offsetting losses against other income therefore causes optimal portfolios of US-American investors trading in a tax-system of the LD type to be diversified, while Canadian or European investors trading in tax-systems of the ND type do not have an incentive to cut gains short and therefore tend to optimally hold less diversified portfolios.

#### 3.5.2 Lower Tax Rate on Individual Income

Having analyzed optimal life cycle portfolio choice for an investor facing a tax rate on individual income of $\tau_i = 35\%$, we next turn to comparative statistics by this rate. Under current tax-law, investors in the 35% tax-bracket enjoy an annual income of at least \$ 349,701. As such investors probably have a wealth level which is substantially above \$ 10,000 and quite likely also above \$ 100,000,[7] we next consider a setting with a tax rate on other income of $\tau_i = 25\%$ here. The annual income of such investors is between \$ 31,851 and \$ 77,100.

Table 3 about here

[7] Exceptions from that rule include highly qualified students leaving university.

Our results in table 3 indicate that decreasing the tax rate on other income to 25% causes the investor to optimally not cut any capital gains short. With the decreased tax rate on other income, the potential advantage from cutting capital gains short also decreases, while the tax-cost of cutting capital gains short remains unchanged. Consequently, as the tax rate on other income decreases, the desire to cut capital gains short decreases.

Even though the opportunity to earn tax rebate payments creates a desire to cut capital gains short, the difference between tax rates on other income and on capital gains in the setting analyzed in this subsection is too small to outweigh the tax-costs from cutting capital gains short.

### 3.5.3 The Exact Basis Rule

In order to keep our optimization problem numerically solvable, we assume the investor's tax-basis to be her average historical purchase price. Even though this average basis-rule is found in many tax-codes around the world, including the Canadian or the Danish tax-codes, it would be interesting to know how the exact basis-rule affects our results. Facing such a treatment of unrealized capital gains, investors no longer have to cut all their capital gains short to regain the opportunity of offsetting potential losses against other income in forthcoming periods. Such investors would optimally sell assets with the highest tax-basis first and not realize all capital gains.

Investors with very low wealth levels would still have to sell most of their equity, while for investors with high wealth levels, a potential tax rebate payment of $ 3,000 would not significantly affect their utility. Consequently, in line with the findings of DeMiguel and Uppal (2005), we believe that the impact of using the exact instead of the average basis-rule should have a minor impact on our results.

Given the fact that even private investors with very low wealth levels facing a tax rate on other income of $\tau_i = 25\%$ optimally do not cut capital gains short given current tax rates and the average basis-rule, investors would not cut capital gains short using the exact basis-rule either.

Solving the optimal consumption portfolio problem with the exact basis rule is a challenging problem since it increases the number of state variables substantially. At present, solutions with more than 10 periods in the one asset-case, or 7 periods in the two-asset case have not yet been computed due to the sharp increase in the numerical complexity of the problem. Since our main goal is to understand the forces driving optimal portfolio choice in the presence of tax rebate payments, we forgo the analysis of the exact basis case.

#### 3.5.4 Non-financial income

In our model, we considered an investor who is not endowed with non-financial income and who has to finance consumption from her financial income. The investor can therefore only increase her wealth level via returns on investments. To analyze the impact of non-financial income, we follow Dammon et al. (2001) and consider a setting where the investor is endowed with labor income of 15% of her initial wealth level in each period. We find that the labor income stream causes the investor's wealth level to increase significantly faster. It therefore holds that ceteris paribus the fraction of losses that can be used against other income decreases faster. Consequently, the incentive to cut capital gains short decreases when the investor is endowed with non-financial income.

### 3.6 Welfare Analysis

To understand whether the \$ 3,000 amount that can be used against other income has an important impact on a private investor's utility, we compute the increase in the initial wealth level at age 20 that an investor in a tax-system of the ST or the ND type has to be endowed with to have the same expected discounted lifetime utility as an investor in the $LD^4$ or the $LD^5$ case.

Table 4 about here

Table 4 shows the results of this welfare analysis for investors subject to tax rates on individual income of 25%, 30%, 35% and 50%, respectively. It indicates that the wealth increase an investor in the $LD^4$ case needs to be endowed with is higher than that of the $LD^5$ investor. This is due to the fact that the fraction of losses that can be offset against other income is higher in the $LD^4$ case. Furthermore, since the ST tax-system is more desirable than the ND tax-system, the wealth increase the investor needs to be given is higher in the ND tax-system compared to the ST system.

For an investor in our base-case parameter setting, her required wealth-increases in the $LD^4$ case compared to the ND and the ST cases are 7.31% and 5.80%, respectively. In case of a tax rate on other income of 50%, these values increase to 39.33% and 36.40%. In case of a tax rate on individual income of 25%, which is probably closest to the current tax rate of an investor with wealth of \$ 10,000 or \$ 100,000, the wealth increase the investor needs to be endowed with is minuscule. This confirms our finding that given current tax rates, the opportunity to offset losses against other income is not a key factor driving private investor's portfolio choice. However, our results also indicate that widening the tax rate differential between the tax rate on other income and the tax rate on capital gains causes the $ 3,000 limit to heavily affect portfolio choice by the desire to cut capital gains short.

# 4 Conclusion

This paper analyzes the optimal dynamic consumption portfolio problem in the presence of capital gains taxes. It explicitly takes into account limited capital loss deduction and the 3,000 dollar amount that can be used against other income. Constantinides (1983) shows that it is optimal to realize capital losses immediately in tax-systems where realized capital gains and losses are subject to the same taxable treatment. We generalize his finding to tax-systems where capital losses can only be applied towards realized capital gains, as well as the one-asset case of tax-systems where capital losses up to a limited amount can also be used against other income. This paper shows that the opportunity to apply limited amounts of capital losses against other income causes investors to hold more diversified portfolios, especially when their total wealth invested is small.

Compared to tax-systems in which capital losses cannot be used against other income, the investment decision becomes more difficult for two reasons. First, the investor has to make a decision on how to use a loss, i.e. whether to potentially postpone the realization of capital gains in order to use it against other income or apply it directly against realized capital gains. Second, in our base-case setting it can be optimal to realize more capital gains than would be required to rebalance the portfolio. In contrast to tax-systems where capital gains and losses are subject to the same taxable treatment and tax-systems where losses can only be used against capital gains, the investor's wealth level in limited deduction systems can have a substantial impact on her portfolio choice. Investors with low wealth levels might want to cut unrealized capital gains short to regain the opportunity of offsetting losses against other income.

However, our base-case parameter setting assumes a significant tax rate differential between the tax rate on other income and the capital gains rate. Since private investors facing such a large tax rate differential are usually endowed with high wealth levels, cutting capital gains short only provides them with a small fraction of potential losses that can be applied against other income. Consequently, such investors optimally do not cut their capital gains short. Investors endowed with a lower wealth level, however, usually face a lower tax rate on individual income such that the tax rate differential is significantly smaller and it is no longer optimal for these investors to cut capital gains short. Hence, given current tax rates, the desire to cut capital gains short in our model is weak. Widening the differential between the tax rate on other income and capital gains, however, would cause investors to cut capital gains short.

In order to keep the optimization problem numerically tractable, our paper's model restricts the number of risky assets to one. It would be interesting to explore optimal tax-timing strategies in the multi-asset case. Analyzing how investors would optimally realize losses in the multi-asset case where it is no longer optimal to realize losses immediately would be a particularly fruitful field for further research. We leave the two-asset case for further research.

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# A Appendix A - Rewritten Optimization Problem

For the numerical solution of the optimization problem (8) to (11), we normalize with beginning-of-period-wealth $W_t$. Let $s_t \equiv \frac{q_{t-1}P_t}{W_t}$ denote the fraction of the investor's beginning-of-period-wealth before trading invested into equity, $\alpha_t \equiv \frac{q_t P_t}{W_t}$ the investor's fraction of beginning-of-period-wealth allocated to equity after trading, $b_t' \equiv \frac{b_t}{W_t}$ the fraction of the beginning-of-period-wealth allocated to risk-free bonds after trading, $c_t \equiv \frac{C_t}{W_t}$ the consumption-wealth-ratio, $p_{t-1}^* \equiv \frac{P_{t-1}^*}{P_t}$ the investors basis-price-ratio, $t_t \equiv \frac{T_t}{W_t}$ the fraction of the investor's beginning-of-period-wealth that is taxable at the capital gains tax rate, $l_{t-1} \equiv \frac{L_{t-1}}{W_t}$ the ratio of the investor's tax loss carry-forward to beginning-of-period-wealth, $d_t \equiv \frac{D_t}{W_t}$ the amount deductible from beginning-of-period-wealth, $g_t \equiv \frac{P_{t+1}}{P_t} - 1$ the capital gain on the stock in period $t$, and

$$R_t \equiv \frac{\alpha_t (1+d)(1+g_t) + b_t' R}{\alpha_t + b_t'} \tag{A.1}$$

the gross nominal return on the investor's portfolio after trading in period $t$ and payment of taxes on dividends and interest, but before payment of capital gains taxes. Defining $v_t(x_t) \equiv \frac{V_t(X_t)}{W_t^{1-\gamma}}$ to be the normalized value function and $\rho_t \equiv \frac{W_{t+1}}{W_t(1+i)}$ to be the investor's real growth of wealth before capital gains taxes, the investor's optimization problem can be rewritten as

$$\begin{aligned} v_t(x_t) = \max_{c_t, \alpha_t . \theta_t} \Bigg[ & U(c_t) + f(t)\beta \mathbb{E}\left[ v_{t+1}(x_{t+1}) \rho_t^{1-\gamma} \right] \\ & + (1 - f(t)) \frac{\beta \left(1 - \beta^H\right)}{1 - \beta} \mathbb{E}\left[\rho_t^{1-\gamma}\right] U(A_H) \Bigg] \end{aligned} \tag{A.2}$$

s.t.

$$1 = \tau_g t_t + \alpha_t + b_t' + c_t - \tau_i d_t \qquad t = 0, \ldots, T \tag{A.3}$$

$$\rho_t = \frac{(1 - \tau_g t_t + \tau_i d_t - c_t) R_t}{1+i} \qquad t = 0, \ldots, T \tag{A.4}$$

$$\alpha_t, b_t' \geq 0 \qquad t = 0, \ldots, T, \tag{A.5}$$

in which $t_t$ and $d_t$ are given by

$$t_t \equiv \max(\delta_t + l_{t-1}; 0) \tag{A.6}$$

$$d_t \equiv \min(-\min(\delta_t + l_{t-1}, 0), m_t), \tag{A.7}$$

$m_t \equiv \frac{M}{W_t}$ and the ratio of realized gains to beginning-of-period-wealth $\delta_t$ and $l_t$ are given by

$$\begin{aligned}\delta_t &\equiv \left(\chi_{\{1>p^*_{t-1}\}}\left(\max\left(s_t-\alpha_t,0\right)+\min\left(s_t,\alpha_t\right)\theta_t\right)+\chi_{\{1\le p^*_{t-1}\}}s_t\right) \\ &\quad \cdot\left(1-p^*_{t-1}\right) &\text{(A.8)}\\ l_t &\equiv \min\left(\delta_t+l_{t-1},0\right)+d_t. &\text{(A.9)}\end{aligned}$$

With $n_t \equiv \max\left(s_t-\alpha_t,0\right)+\min\left(s_t,\alpha_t\right)\theta_t$, the basis-price-ratio $p^*_t$ is given by

$$p^*_t=\begin{cases}\frac{(s_t-n_t)p^*_{t-1}+\max(\alpha_t-s_t,0)+\min(s_t,\alpha_t)\theta_t}{\alpha_t(g_t+1)} & \text{if } p^*_{t-1}<1\\ \frac{1}{g_t+1} & \text{if } p^*_{t-1}\ge 1.\end{cases} \tag{A.10}$$

At time $T$, the investor's normalized value function takes the value

$$v_T(x_T)=\max_{c_T,\alpha_T,\theta_T} U\left(c_T\right)+\frac{\beta\left(1-\beta^H\right)}{1-\beta}\mathbb{E}\left[\rho_T^{1-\gamma}\right]U\left(A_H\right) \tag{A.11}$$

due to the forgiveness of capital gains upon death. The vector $x_t$ of state variables at time $t$ of the normalized optimization problem is given by

$$x_t=[p^*_{t-1},s_t,l_{t-1},m_t]. \tag{A.12}$$

The recursive form of the optimization problem is given by

$$\begin{aligned}v_t\left(x_t\right) &= \max_{c_t,\alpha_t,\theta_t} U\left(c_t\right)+\mathbb{E}\Bigg[\rho_t^{1-\gamma}\Bigg(f(t)\beta v_{t+1}\left(x_{t+1}\right)\\ &\quad +\left(1-f(t)\right)\beta\frac{1-\beta^H}{1-\beta}U\left(A_H\right)\Bigg)\Bigg], &\text{(A.13)}\end{aligned}$$

subject to (A.3) to (A.10) with terminal condition

$$v_T(x_T)=\max_{c_T,\alpha_T,\theta_T} U\left(c_T\right)+\frac{\beta\left(1-\beta^H\right)}{1-\beta}\mathbb{E}\left[\rho_T^{1-\gamma}\right]U\left(A_H\right). \tag{A.14}$$

For values of state variables that are not on the grid, we perform cubic spline interpolation. The integral in the expectation of the investor's utility in equation (A.13) is computed using Gaussian quadrature.

| Description | Parameter | Value |
|---|---|---|
| Risk-aversion | $\gamma$ | 3 |
| Length of investment horizon | $T$ | 80 |
| Number of years annuity beneficiary | $H$ | 60 |
| Annual utility discount factor | $\beta$ | 0.96 |
| Post-tax dividend rate | $d$ | 1.3% |
| Expected pre-tax capital gains rate stock | $\mu$ | 7% |
| Standard deviation of capital gains rate stock | $\sigma$ | 20.7% |
| Post-tax interest payment of bond | $r$ | 3.9% |
| Annual inflation rate | $i$ | 3.5% |
| Tax rate on interest and dividend income | $\tau_i$ | 35% |
| Tax rate on realized capital gains | $\tau_g$ | 20% |

Table 1: This table reports our parameter values used in the base-case.

| Panel A - Age 30 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha_t$ in % | | | | $\theta_t$ in % | | $p^*_{t-1}$ | | | | $F^{(I)}_\Sigma$ | |
| Perc. | LD$^4$ | LD$^5$ | ND | ST | LD$^4$ | LD$^5$ | LD$^4$ | LD$^5$ | ND | ST | LD$^4$ | LD$^5$ |
| 1 | 47.1 | 32.6 | 26.6 | 30.4 | 100 | 0 | 0.552 | 0.552 | 0.162 | 0.162 | 100 | 38 |
| 10 | 49.4 | 37.4 | 28.6 | 33.4 | 100 | 0 | 0.736 | 0.676 | 0.289 | 0.289 | 100 | 53 |
| 50 | 50.9 | 40.7 | 33.6 | 37.9 | 100 | 32 | 0.955 | 0.883 | 0.539 | 0.533 | 100 | 84 |
| 90 | 52.9 | 42.4 | 43.7 | 43.9 | 100 | 75 | 1.238 | 1.178 | 0.923 | 0.912 | 100 | 100 |
| 99 | 54.5 | 45.3 | 47.0 | 46.8 | 100 | 83 | 1.651 | 1.500 | 1.238 | 1.238 | 100 | 100 |
| Mean | 51.1 | 40.2 | 35.3 | 38.1 | 100 | 41 | 0.974 | 0.918 | 0.580 | 0.580 | 100 | 79 |
| Std | 1.5 | 2.4 | 5.4 | 4.1 | 2 | 25 | 0.189 | 0.182 | 0.254 | 0.254 | 1 | 17 |
| Panel B - Age 60 | | | | | | | | | | | | |
| 1 | 41.7 | 30.3 | 25.8 | 26.6 | 8 | 0 | 0.552 | 0.064 | 0.009 | 0.009 | 81 | 44 |
| 10 | 44.8 | 33.4 | 30.0 | 32.2 | 83 | 0 | 0.736 | 0.215 | 0.031 | 0.031 | 93 | 58 |
| 50 | 47.0 | 39.0 | 42.7 | 43.2 | 100 | 0 | 0.955 | 0.612 | 0.129 | 0.129 | 100 | 73 |
| 90 | 52.4 | 45.8 | 50.8 | 50.6 | 100 | 22 | 1.238 | 0.955 | 0.509 | 0.503 | 100 | 88 |
| 99 | 54.5 | 49.8 | 52.2 | 52.0 | 100 | 53 | 1.651 | 1.238 | 0.955 | 0.955 | 100 | 98 |
| Mean | 47.8 | 39.2 | 41.4 | 42.1 | 95 | 5 | 0.968 | 0.610 | 0.210 | 0.202 | 98 | 73 |
| Std | 3.0 | 4.4 | 7.6 | 6.9 | 16 | 12 | 0.187 | 0.295 | 0.217 | 0.215 | 4 | 11 |
| Panel C - Age 90 | | | | | | | | | | | | |
| 1 | 37.7 | 32.5 | 30.8 | 32.1 | 0 | 0 | 0.117 | 0.006 | 0.001 | 0.001 | 79 | 45 |
| 10 | 40.8 | 38.1 | 36.6 | 38.7 | 0 | 0 | 0.256 | 0.027 | 0.005 | 0.005 | 88 | 59 |
| 50 | 46.8 | 52.8 | 57.2 | 57.6 | 0 | 0 | 0.551 | 0.132 | 0.036 | 0.036 | 96 | 75 |
| 90 | 57.0 | 66.3 | 70.9 | 70.9 | 0 | 0 | 0.939 | 0.509 | 0.231 | 0.289 | 100 | 89 |
| 99 | 65.6 | 72.5 | 73.3 | 73.3 | 0 | 0 | 1.238 | 0.955 | 0.831 | 0.820 | 100 | 98 |
| Mean | 48.0 | 52.5 | 55.4 | 56.1 | 0 | 0 | 0.578 | 0.209 | 0.103 | 0.103 | 95 | 74 |
| Std | 6.3 | 10.3 | 12.3 | 11.7 | 0 | 0 | 0.260 | 0.217 | 0.167 | 0.165 | 5 | 11 |

Table 2: Simulation Analysis - Base Case Parameter Setting: This table shows the evolution of the investor's optimal investment strategy (her optimal equity exposure $\alpha_t$ as well as her optimal cutting of gains $\theta_t$ in tax-systems of the LD type) and her basis-price-ratio before trading $p^*_{t-1}$ in tax-systems of the LD, the ND and the ST types when the investor's tax-rate on interest, dividends and other income is $\tau_i = 35\%$. $F^{(I)}_\Sigma$ denotes the fraction of losses the investor has used against other income until time $t$ in those cases where the investor has made use of a tax loss. The values presented here are the results of 50,000 simulations on the optimal paths. The columns marked LD$^4$ refer to an investor with an initial wealth level of \$ 10,000 at age 20, the columns marked LD$^5$ refer to an investor with an initial wealth level of \$ 100,000.

Panel A - Age 30

| | $\alpha_t$ in % | | | | $\theta_t$ in % | | $p^*_{t-1}$ | | | | $F^{(I)}_\Sigma$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Percentile | LD$^4$ | LD$^5$ | ND | ST | LD$^4$ | LD$^5$ | LD$^4$ | LD$^5$ | ND | ST | LD$^4$ | LD$^5$ |
| 1 | 27.0 | 26.9 | 23.9 | 26.3 | 0 | 0 | 0.162 | 0.162 | 0.162 | 0.162 | 100 | 79 |
| 10 | 29.8 | 29.5 | 26.0 | 28.1 | 0 | 0 | 0.289 | 0.289 | 0.289 | 0.289 | 100 | 95 |
| 50 | 33.9 | 33.6 | 29.7 | 33.4 | 0 | 0 | 0.533 | 0.534 | 0.539 | 0.533 | 100 | 100 |
| 90 | 40.1 | 40.1 | 40.1 | 40.2 | 0 | 0 | 0.913 | 0.913 | 0.935 | 0.912 | 100 | 100 |
| 99 | 43.1 | 43.2 | 43.4 | 43.2 | 0 | 0 | 1.238 | 1.238 | 1.238 | 1.238 | 100 | 100 |
| Mean | 34.5 | 34.2 | 31.5 | 33.6 | 0 | 0 | 0.581 | 0.580 | 0.582 | 0.579 | 100 | 98 |
| Std | 3.6 | 3.9 | 5.2 | 4.5 | 0 | 0 | 0.253 | 0.255 | 0.255 | 0.254 | 0 | 4 |
| Panel B - Age 60 | | | | | | | | | | | | |
| 1 | 25.3 | 25.1 | 22.6 | 24.0 | 0 | 0 | 0.009 | 0.009 | 0.009 | 0.009 | 100 | 79 |
| 10 | 30.0 | 29.5 | 26.7 | 28.2 | 0 | 0 | 0.031 | 0.031 | 0.031 | 0.031 | 100 | 94 |
| 50 | 40.0 | 40.0 | 39.0 | 39.8 | 0 | 0 | 0.128 | 0.127 | 0.133 | 0.132 | 100 | 99 |
| 90 | 47.0 | 47.0 | 47.0 | 47.0 | 0 | 0 | 0.503 | 0.495 | 0.530 | 0.516 | 100 | 100 |
| 99 | 48.1 | 48.2 | 48.2 | 48.1 | 0 | 0 | 0.955 | 0.955 | 0.955 | 0.955 | 100 | 100 |
| Mean | 39.2 | 39.1 | 37.8 | 38.6 | 0 | 0 | 0.207 | 0.205 | 0.216 | 0.212 | 100 | 98 |
| Std | 6.4 | 6.5 | 7.6 | 7.0 | 0 | 0 | 0.215 | 0.213 | 0.223 | 0.218 | 0 | 4 |
| Panel C - Age 90 | | | | | | | | | | | | |
| 1 | 28.0 | 27.5 | 26.7 | 27.3 | 0 | 0 | 0.001 | 0.001 | 0.001 | 0.001 | 100 | 80 |
| 10 | 35.0 | 34.4 | 32.0 | 33.9 | 0 | 0 | 0.004 | 0.004 | 0.005 | 0.005 | 100 | 94 |
| 50 | 51.3 | 51.2 | 52.7 | 53.1 | 0 | 0 | 0.033 | 0.032 | 0.038 | 0.037 | 100 | 99 |
| 90 | 64.8 | 64.8 | 67.5 | 67.6 | 0 | 0 | 0.252 | 0.248 | 0.303 | 0.295 | 100 | 100 |
| 99 | 70.2 | 70.2 | 71.0 | 71.0 | 0 | 0 | 0.796 | 0.794 | 0.847 | 0.831 | 100 | 100 |
| Mean | 50.6 | 50.4 | 51.1 | 51.8 | 0 | 0 | 0.094 | 0.092 | 0.107 | 0.105 | 100 | 98 |
| Std | 10.8 | 11.0 | 12.8 | 12.2 | 0 | 0 | 0.157 | 0.154 | 0.172 | 0.159 | 0 | 4 |

Table 3: Simulation Analysis - $\tau_i = 25\%$: This table shows the evolution of the investor's optimal investment strategy (her optimal equity exposure $\alpha_t$ as well as her optimal cutting of gains $\theta_t$ in tax-systems of the LD type) and her basis-price-ratio before trading $p^*_{t-1}$ in tax-systems of the LD, the ND and the ST types when the investor's tax-rate on interest, dividends and other income is $\tau_i = 25\%$. $F^{(I)}_\Sigma$ denotes the fraction of losses the investor has used against other income until time $t$ in those cases where the investor has made use of a tax loss. The values presented here are the results of 50,000 simulations on the optimal paths. The columns marked LD$^4$ refer to an investor with an initial wealth level of \$ 10,000 at age 20, the columns marked LD$^5$ refer to an investor with an initial wealth level of \$ 100,000.

| $\tau_i$ in percent | 25 | 30 | 35 | 50 |
|---|---|---|---|---|
| $LD^4$ vs ND | 1.58 | 2.34 | 7.31 | 39.33 |
| $LD^5$ vs ND | 1.44 | 1.78 | 2.33 | 10.47 |
| $LD^4$ vs ST | 0.49 | 1.08 | 5.80 | 36.40 |
| $LD^5$ vs ST | 0.35 | 0.53 | 0.89 | 8.15 |

Table 4: Welfare Analysis: This table shows the percentage increase in wealth a 20 year old investor in the ND or the ST tax-system needs to be endowed with to end up with the same level of expected lifetime utility as an investor a tax-system of the LD type who is endowed with an initial wealth level of 10,000 ($LD^4$) or 100,000 ($LD^5$) dollars, respectively. We consider an investor without unrealized capital gains, losses, or initial tax loss carry-forward.

## Optimal Investment Policy

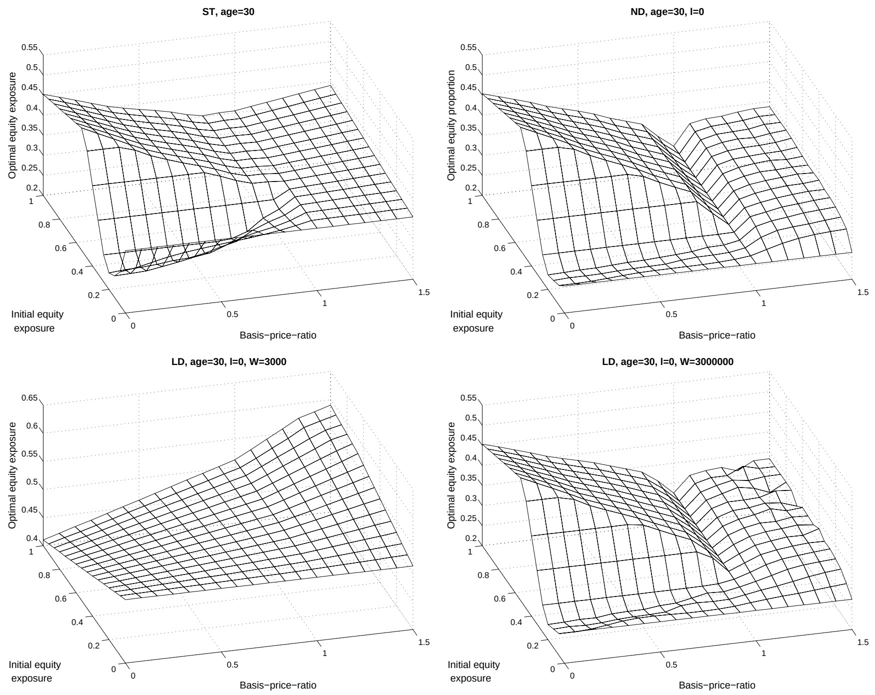


Figure 1: This figure shows the relation between the investor's optimal equity exposure and her initial equity exposure as well as her initial basis-price-ratio for an investor at age 30 who is not endowed with an initial tax loss carry-forward. The upper left graph shows the optimal equity exposure of an investor in an ST tax-system, and the upper right graph depicts the optimal equity exposure for an investor trading in an ND tax-system. The lower graphs show the optimal equity exposure of an investor in a tax-system of the LD type who is endowed with an initial wealth of \$ 3,000 (lower left graph) and an initial wealth of \$ 3,000,000 (lower right graph).

## Optimal Investment Policy and Wealth Level

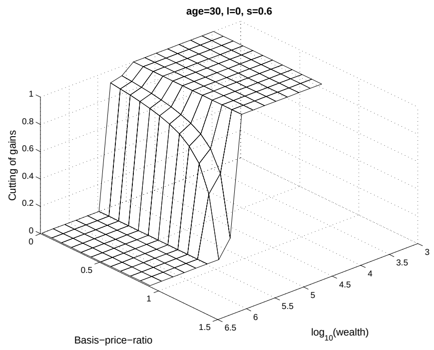


Figure 2: This figure depicts how the optimality of cutting unrealized capital gains short depends on her basis-price-ratio and her initial wealth level. We consider an investor who is not endowed with an initial tax loss carry-forward $l = 0$ and whose initial equity exposure is $s = 60\%$.

**Optimal Investment Policy with Tax Loss Carry-Forward**

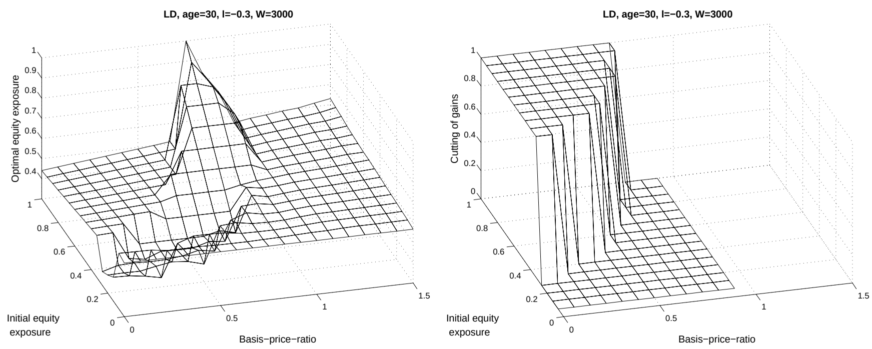


Figure 3: This figure depicts the relation between the investor's equity exposure (left graph) as well as her optimal cutting of unrealized capital gains (right graph) depending on her initial equity exposure and her initial basis-price-ratio. These graphs consider an investor at age 30 who is endowed with an initial tax loss carry-forward of $l = -30\%$ of her initial wealth and a total wealth of \$ 3,000.

## Effective Tax Rate

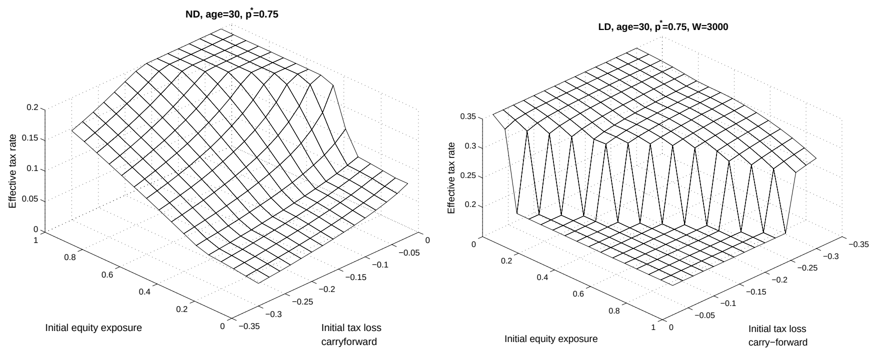


Figure 4: This figure depicts the effective tax rate applicable to a tax loss carry-forward that makes an investor indifferent between carrying a tax loss carry-forward over and receiving an immediate tax rebate payment at that tax rate. We consider an investor with an initial basis-price-ratio of $p^* = 0.75$, indicating that the investor is facing unrealized capital gains. Both graphs show the impact of the investor's initial equity exposure and the level of her tax loss carry-forward on her effective tax rate at age 30, the left in an ND tax-system, and the right graph in an LD tax-system for an investor endowed with an initial wealth level of $ 3,000.