Why are British Premium Bonds so successful?
The effect of saving with a thrill

Abstract
The British Premium Bond, which offers a monthly uncertain return solely based on a lottery, is an immense success. Why? We find that the bond bears relatively low risk in terms of CARA and CRRA utility. Since prizes are tax-free, the higher an individual's tax bracket, the more it pays to invest in the lottery bond. However, we demonstrate that the CARA and CRRA coefficients (before and after taxes) do not directly influence sales of the Premium Bond. Rather, our ARIMA model strongly suggests that prize skewness and the maximum holding amount are the salient influencing factors.

J.E.L. Classification: G12, G11.

Keywords: Premium Bond, lottery bond, risk tolerance, skewness.

Introduction

Can saving money, without risking the principal, become an adventure? Looking at ordinary savings accounts, one readily answers no. An investor pays an amount of money into a bank account and gets fixed interest payments: a humdrum but safe way of investing. One very popular way of getting a thrill is gambling as people are always happy about winning a prize. Centuries ago, financial products were invented to capitalize on people's fascination for gambling. The idea features saving money with a lottery to make things more exciting. As a result, the issuers usually enjoyed significantly higher sales and profits. Nowadays, lottery bonds or lottery-linked deposit accounts (LLDAs) are available worldwide. One very successful example is the British Premium Bond. Harold MacMillan, Chancellor of the British Exchequer, initially launched the British Premium Bond in November 1956. After decades of steadily increasing sales, particularly in the last 7 years commencing in 2000, the Premium Bonds sky-rocketed. Currently, 23 million people in Great Britain have invested more than £33 billion in Premium Bonds. What makes these so successful? We provide an answer to this question after having hand-collected the data over a period of fifty years. Because of its longevity, the British Premium Bond is perfect for an empirical analysis on how a successful lottery bond can work.

Much research has already been done on analysing individual risk preferences. Often the central question is what risk preferences do individuals exhibit in certain situations and when do they accept bets with even negative expected returns? While many studies use surveys, e.g. Donkers, Melenberg and Van Soest (2001), others analyse large data samples from TV game shows, e.g. Beetsma and Schotman (2001), or horse races like Jullien and Salanie (2000). Lottery bonds can also be analysed in this context. As these investments are not traded in an artificial environment, it makes them particularly interesting for empirical studies. Guillen and Tschoegl (2002) describe numerous examples of LLDAs with focus on examples located in Latin America. They conclude that these accounts are apparently more a marketing device than a source of funds cheaper than savings deposits. Ukhov (2005) studies the relationship between investor risk preference and asset returns of Russian lottery bonds. He analyses time variations in the risk preferences between 1889 and 1904. Green and Rydqvist (1997, 1999) study Swedish government lottery bonds whose coupon payments are determined by a lottery. They evaluate the rewards of bearing extra lottery risk, finding that prices appear to reflect this risk. They also report that variance reduces lottery prices. Florentsen and Rydqvist (2002), analysing the pricing of Danish lottery bonds focusing on tax-based explanations of abnormal ex-day returns, find that prices fall by more than the lottery mean and also conclude that investors do not enjoy this lottery. Pfiffelmann (2006) analyses the optimal design for LLDAs. Guillen and Tschoegl (2002) also state that skewness of returns is a feature to maintain investors' interest in the LLDA. Many studies on gamblers' risk attitudes have discussed the importance of the third moment. Golec and Tamarkin (1998) point out that not only mean and variance explain gambling behaviour but also skewness of the returns. Garrett and Sobel (1999) find evidence for the relevance of skewness by examining United States lotteries. Bhattacharyya and Garrett (2006) empirically find that the expected return from a lottery game is a decreasing and convex function of the skewness of the lottery game.

This paper is organised as follows. First, we explain the basic design of the bond. Then, we compute the degrees of risk aversion and risk seeking an investor needs to exhibit in order to prefer Premium Bonds. For this purpose, we apply the classical Arrow-Pratt Constant Absolute Risk Aversion (CARA) and Constant Relative Risk Aversion (CRRA) approaches to back out the indifference degree of risk tolerance. As the investment alternatives are taxed differentially, individual income taxes play a key role. Additionally, we compare our results to an approximation by Pratt (1964). We focus first on a monthly investment period for each month and tax class over the full fifty years time frame. Then, we study longer investment periods of five, ten and twenty years. Finally, we discuss important factors influencing the success of Premium Bonds. We scrutinize whether risk preference and sales are correlated. We also focus on prize skewness as a major influencing factor. In order to detect short-run and long-run relationships, we conduct Granger Causality Tests and Johansen Cointegration Tests on multiple potential influencing factors. Finally, we present an ARIMA regression model to show that skewness and maximum holding are indeed the factors that actually encourage sales.

1. History of the Premium Bond and its Characteristics

The Premium Bond is issued by National Savings and Investments (NS&I), which has been a government department since 1969. It follows two principles: to provide a secure vehicle for people to save, backed by the government on one hand and to provide the exchequer with a source of funding on the other (i.e. public borrowing) (see NS&I, 2005). Launched in 1956, the Premium Bond has been slowly expanding over 35 years. Since 1991, sales have been steadily increasing. Today, this bond is one of the most important investment products in Great Britain for households and it is NS&I's most successful asset. Figure 1 shows how the monthly net contribution (= sales – repayments) has developed over time.

Place figure 1 here

Premium Bonds definitely enjoy the highest popularity since 23 million Britons (about 40% of the population) own these. Table 1 illustrates the rapid growth of the last five years.

Place table 1 here

The initial purpose of the Premium Bond was to control inflation and to encourage more people after World War II to save money. For almost thirty years (1950s – 1980s) gambling this way was advertised as a fun way of saving and investing money. The National Lottery was then launched years later in November 1994. Since the 1990s, NS&I changed its marketing strategy and emphasised that Premium Bonds are a serious way of investing money, leading to a huge escalation in sales (NS&I, 2006).

The basic design of the bond is quite simple and has not been altered since its conception: any British citizen aged 16 and over can buy Premium Bonds. It is not possible to hold them jointly and they are not transferable to another person. Currently, the minimum amount invested is £100 or £50 with a monthly standing order. Unlike a common deposit account, the total interest payments per month are subject to a lottery. There are no additional interest payments. The fee for participating in the prize draws is just the forgone interest payment of an alternative investment. For each single pound invested, there is one chance to win. Currently the maximum amount a person can invest is £30,000. For example, if someone buys Premium Bonds worth £3,000, he or she has 3,000 chances to win. Each bond has exactly the same chance, making time of purchase irrelevant. The prize draws are carried out at the beginning of each month by a sophisticated computer system, which NS&I calls ERNIE (Electronic Random Number Indicator Equipment). The odds of winning a prize are currently 24,000 to 1 (September 2006). This means that an investor holding £24,000 can expect to win once per month on average. Of course, this is not guaranteed. After several changes, the prizes have been spread from £50 up to £1 million. The total number of prizes per month is calculated by the total number of eligible bond units divided by the odds. The total value of all prizes of a draw is determined by the interest rate that is announced in advance. NS&I can arbitrarily change this rate. On their official web page, NS&I states that 87% of the prize fund is allocated to the lower prize band; 6% to the medium band; and 7% to the higher prize band. Table 2 illustrates what a typical prize draw looks like.

Place table 2 here

One special feature of the Premium Bond is that all prizes are tax-free, making them even more attractive for potential investors. Unlike a regular lottery, the initial investment is not used up. Moreover, a bond holder can always get the principal refunded at any time. This advantage, plus the maximum holding stipulation, controls the risk of addiction and possible financial ruin.

2. Classical Risk Tolerance Analysis

2.1 Research Method and Preliminary Considerations

In this section, we now analyse the extent to which an investor needs to be risk averse or risk seeking in order to consider Premium Bonds as a utility maximising investment. A classical approach is the expected utility theory operationalised by Arrow's (1965) and Pratt's (1964) risk measure.

$$\text{Constant Absolute Risk Aversion (CARA):} \quad u(x) = -\frac{e^{-\alpha \cdot x}}{\alpha} \qquad (1)$$

$$\text{Constant Relative Risk Aversion (CRRA):} \quad u(x) = \frac{x^{1-\alpha}}{1-\alpha} \qquad (2)$$

In the above equations, x stands for the amount of money, α for the individual risk preference and $u(x)$ for the utility of x. e is the base of the natural logarithm. In order to obtain the indifference level of risk tolerance, we iteratively calculate the coefficient α, which leads to the same utility of a "risky" Premium Bond and a certain alternative investment. For comparison, we compute both the constant absolute risk aversion and the constant relative risk aversion.

$$E\left[u(x)\right] = \sum_{i=1}^{n} p_i \cdot u(x_i), \; p_i = \frac{c_i}{t \cdot o} \qquad (3)$$

The expected utility of the Premium Bond for a month´s draw is obtained as follows. We calculate the utility of each prize of a draw $u(x_i)$ including the case that nothing is won. Utility components are weighted with the specific probability of occurrence p_i. To calculate these probabilities, we divide the number of prizes in each prize class c_i (e.g. 78 times

£10,000) by the total number of prizes of this draw *t* (e.g. November 2006: 1,347,081). This likelihood is divided by the odds *o* to obtain the probability p_i that £1 wins exactly this prize. Monthly interest payments determine the utility of a certain investment. By iterative calculation, we obtain values for α (CARA, CRRA). An individual investor exhibiting this indifference risk coefficient would be indifferent between the two alternatives. As α is a small number and very sensitive with respect to the accuracy of the interpolation, we perform our calculations with 300 decimal places. As figure 2 shows, positive (negative) values of α indicate risk aversion (risk seeking) across time. A zero value means risk neutrality. Investors who are less risk averse or more risk seeking than the indifference level depicted will choose the Premium Bond since this maximizes their utility.

Place figure 2 here

Next, we need to specify reference investments. As we try to employ the longest data record possible, the official Bank of England's (BoE) rate matches this objective nicely. While we are aware that a retail investor cannot invest in a bond delivering the BoE rate, most bonds in the UK should be linked to this rate to a greater or lesser extent. In order to understand how Premium Bonds perform in comparison to a product an investor can actually purchase, we choose to pick the Income Bond delivering monthly interest payments. This investment, issued as well by NS&I, implies that there will not be a differing issuer's risk premium. Since NS&I is secured by the government, the products are essentially risk-free. Premium Bonds and Income Bonds are similar in terms of the initial investment, the monthly payout structure, the option to withdraw the safe capital at any time and the infinite time to maturity. However, the Income Bond's monthly interest payment is certain, and the interest rate is higher but subject to income taxation.

For our analysis, the margin between the interest rate of the Premium Bond and of other investments is crucial. High expected returns of the lottery bond compared with other investments can encourage even risk averse investors to buy it. Figure 3, illustrating the corresponding time series, shows how the interest rates of the observed investments have changed in the last fifty years.

Place figure 3 here

Another key element is taxation. As previously noted, Premium Bonds enjoy tax exemption which makes them more attractive for investors. For example, the current 3.40% interest rate (12.1.2007) is equivalent to 5.66% for a higher income taxpayer, and to 4.24% for a basic taxpayer. Therefore, considering after-tax returns, it is possible that Premium Bonds outperform other risk-free investments. Table 3 shows current income tax rates based on the UK tax legislation.

Place table 3 here

Due to personal allowances, (e.g. 2006-07 £5,035), we also include the case that an investor is not liable for taxation. Since our analysis covers fifty years, we always try to apply the tax rates valid for that year in consideration. In essence, while the tax classes have not changed, the tax rates have been subject to several changes. We were able to obtain the tax rates from the year 1973 until now (Adam and Browne, 2006). For previous years, we presume that the tax rates were 50% (higher rate), 30% (basic rate) and 10% (starting rate). After studying the UK surtax rates from 1948 to 1973, these rates seem reasonable. We assume that in the higher rate tax bracket, an investor needed to pay the highest possible rate within this band which was up to 75,83% in the 1970s. Also note that the starting tax rate was raised only in a few years.

Checking the overall taxpayer distribution for the UK, we find that in 2005-06, 10.8% of all taxpayers were attributed to the higher rate tax, 75% to the basic rate tax and 14.2% to the starting rate tax (Adam and Browne, 2006). This distribution was also valid for the time period until 1994. From 1978 to 1990, there was no starting tax rate and therefore more than

93% of all taxpayers were basic rate taxpayers. Since 23 million Britons own Premium Bonds, which representing about 38% of the recent population, it is reasonable to assume that most bond holders pay the basic rate. With regard to the Premium Bond, on average, each investor possesses about £1,400 in Premium Bonds (calculated from November 2006 figures according to the NS&I Media Centre). In May 2006, NS&I published that more than 1.5 million people have deposited £5,000 or more, accounting for about 6.5% of all bond holders. The maximum investment of £30,000 is held by 300.000 people, 1.3% of all investors.

2.2 Data

The hand-collected Premium Bond data comprise 592 monthly prize draws from its first draw in June 1957 up to September 2006. For each month, we have information on the number of prizes (individual and total), the value of the prizes, the odds of winning, the maximum holding and the interest rate on which the prize distribution is calculated. Furthermore, we also gained access to sales records and repayments. From October 1969 to April 2006, there were 439 monthly observations. Before this time span, only annual figures were available. Therefore, when working with sales data, we confine our analysis to the period from 1969 to April, 2006. The Income Bond data contain all the interest rates commencing in July 1982, when the bond was initially launched, until September, 2006. In order to make the Premium Bond and the Income Bond comparable, we identify the Income Bond interest rate at the beginning of each month, yielding 292 observations. We also collect the official Bank of England base rate at the beginning of each month from March 1957 to September 2006 (595 observations). Additionally, for a long-run analysis, we use 240 Bank of England UK nominal spot curves at the month’s beginning (Jan 1979 - Dec 1998).

2.3 Short-run Risk Coefficients

Starting off with a myopic approach, we compute the value of α, considering taxation for each month from the first draw in October 1957 until September 2006. Assuming that an investor has deposited £1 and does not intend to get his or her principal refunded within or right after the time period, then his only concern is the monthly lottery winnings or the interest payments. Furthermore, our investor possesses no additional wealth which influences the CRRA utility function. This simple initial setting will be later extended. By iteration, we can calculate the indifference risk coefficient α. Knowledge of this figure over the whole time frame shows an investor ex post if the decision in favour of the Premium Bond has been utility maximizing or not, with respect to his individual degree of risk tolerance. By tracking the α- values over the full time period, we can assess which individual risk preferences investors needed to exhibit in order to consider Premium Bonds as an attractive way of saving money and how these changed over the past decades. In this context, we also compute the monthly α-values by using Pratt's approximation formula (1964), to judge how well the approximation performs.

2.3.1 Results with the Bank of England Base Rate as Reference

Since this is the lengthiest data record available, we calculate the α- values for an investor who decides between Premium Bonds and the virtual investment which delivers interest payments linked to the official Bank of England base rate. Table 4 presents the summary statistics.

Place table 4 here

Our calculation results are based on 592 values in the higher, basic and no tax classes. The starting rate tax class only comprises 388 observations because in some years no such tax was raised. Before 1973, we do not have extant information whether a starting tax rate existed or not. Thus, we decided to assume a constant 10% tax rate, helping to draw conclusions in which situation a starting rate taxpayer would have been over the whole period. Figure 4 displays the time series of the CARA risk coefficients for higher and basic rate taxpayers.

Place figure 4 here

In years like 1977, the combination of a Premium Bond interest rate exceeding the BoE rate and the advantage that prizes are tax-free increased the expected utility to such a degree that even a risk averse investor with a CARA $\alpha < 0.086$ would prefer the risk-carrying Premium Bond. Generally, for higher income taxpayers, an investment in the lottery bond becomes a lot easier attractive in terms of risk tolerance. The lower the individual taxation of an investor, the less risk averse or more risk seeking he or she needs to be. We further observe, moving towards the present, that volatility decreases and the trend goes towards risk neutrality due to a better controlled and thus relatively constant margin of interest. Premium Bonds may still be interesting even for risk averse higher income taxpayers, but their degree of risk tolerance needs to be located close to risk neutrality. Although all the other taxpayers required some risk seeking traits, the values of the CARA α have been surprisingly close to risk neutrality in the last few years.

When looking at the results of the constant relative risk aversion analysis, note that personal wealth has not yet been included.

Place figure 5 here

The relative risk aversion parameters α are scattered from 0.134 to -0.108. The calculation shows that, over time, the risk coefficients changed frequently depending on the interest spread between the Premium Bond and the Bank of England rate. While volatility was great until the mid-1990's, it has steadily decreased to the present. In general, the risk coefficients of the separate tax classes follow the same pattern. Since the Premium Bond interest rate has been adjusted regularly and kept on a fair level compared to the official base rate, the risk coefficients now lie close to zero. In the last two years since 2004, the extreme parameter values α lay between 0.01226 and -0.05459.

2.3.2 Results with the Income Bond as Reference

After this first examination, we now compare the results with a product which can be actually purchased – the Income Bond. Due to the aforementioned shortened data record, there are no conclusions possible before 1982. Table 5 presents the summary statistics.

Place table 5 here

The CARA risk coefficients vary between –0.00002249 and 0.01323053. In terms of relative risk aversion, we observe values between -0.07845 and 0.05732. Plotting linear trend lines makes clear that the level of risk tolerance for high income taxpayers tended towards risk neutrality. On the other hand, the required degree of risk loving for basic and starting rate taxpayers also decreased in favour of investing in Premium Bonds. In general, both indifference lines converged more and more to the risk neutrality level. In order to prefer Premium Bonds to Income Bonds, one needed to be somewhat less risk averse or a bit more risk seeking compared to the BoE base rate. The mean coefficient for higher rate taxpayers with the Income Bond as reference is 0.020219. Calculating the mean with the BoE rate as reference for the same period produces 0.020494. The expected Premium Bond payouts each month were, on average, 69.62% of the theoretical BoE interest payments, but only 68.68% of the Income Bond interest payments. This led to a somewhat lower indifference level of risk aversion when having Income Bonds as reference. These two ratios also clearly show that the Premium Bond yields substantially lower expected payouts than the risk-free alternatives. The internal bias due to the tax-relief is also an important consideration.

2.3.3 Inclusion of Personal Wealth and Higher Investment Amounts

We now extend our initial calculations by assuming that an investor possesses wealth in addition to an investment in Premium Bonds or in an alternative one. As mentioned before, the current average amount invested in Premium Bonds is about £1,400. Thus, we no longer calculate the CRRA indifference risk coefficients with £1, but with a £1,400 deposit. Since we lack data on how the average amount invested has changed since 1957, we compute equivalent values for each month by adjusting them with the respective retail price index for each month. The basis for this retail price index is 100 in January 1987 (National Statistics, 2006). Hence, for example, £1,400 in August 2006 is equivalent to £83.6 in July 1957. This method makes sure that the assumed money invested is always consistent with the current price level. The situation is similar to the first setting. An investor only compares the utility of the payout of the investments in each month. But, we also now take into account the utility of additional wealth. As a proxy, we use personal income per year, showing the effects on utility if one had a certain percentage of his/her yearly income invested. For each tax class, we assume a representative amount of wealth. Inferences on that are drawn by analysing the income tax allowances and the bands for each tax class. The following values are used for our estimation: yearly income of a person who is not liable to tax £2,518, for a starting rate taxpayer £6,110[1], for a basic rate taxpayer £21,685, and finally £47,035 for a higher rate taxpayer. Again, the income in each tax band is adjusted by the retail price index. In this setting, we can find the indifference coefficient for each month under the assumption that the situation was always the same compared to August 2006. Table 6 depicts these results.

Place table 6 here

[1] Calculation: allowance (person under 65 years) + mean of tax band (here 0-£2,150) = £5,035+(£2,150/2) = £6,110.

We observe, in line with our previous findings, that all investors except the higher income taxpayers needed to be risk loving. The pattern of the indifference lines is equivalent to the simple case. However, now the values are quantitatively larger, which means that the CRRA risk coefficients are now wider than in the previous case. The minimum value is -0.3153 and the maximum value is 3.5985. This indicates that, on the one hand, sometimes even quite risk averse investors have been indifferent between Premium Bonds and a risk-free investment which yielded the BoE interest rate. On the other hand, starting rate and non-taxpayers had to be more risk-loving. We observe that, on average, the reaction of the risk coefficients is more extensive the higher the taxation is (see table 7).

Place table 7 here

In the next step, we want to learn how the risk coefficients change when we assume that an investor always kept the highest possible investment. We still use the same time adjusted wealth as before for the separate taxpayer groups. In the past fifty years, the maximum holding was increased in 5 steps from £500 to £30,000, as following:

Place table 8 here

Table 9 presents the results of the calculation.

Place table 9 here

Assuming an investor always had the maximum holding invested, we find that the risk tolerance had to be more risk neutral or risk loving. Furthermore, the distribution of the values is not as broad as in the previous case with a relatively small amount invested. Due to the higher stakes, it is logical that investors have to take on more risks. However, some risk averse higher income taxpayers still preferred the lottery bond to a risk-free investment.

To check how the result of the comparison with the official Bank of England base rate applies to an actual product, we calculate once again the scenario respectively for the Income Bond.

Again, we assume the time adjusted wealth and furthermore a time adjusted £1,400 investment. The results in table 10 are, as expected, quite similar to our previous findings.

Place table 10 here

We see that the indifference risk coefficients move within a narrower margin. High income taxpayers with an individual risk tolerance coefficient bigger than 0.22778 did not opt for the Premium Bond. On the other hand, only a slight degree of risk-seeking (-0.10132) prompted non-taxpayers to try their luck with the Premium Bond.

2.4 Long-term Analysis

We have thus far analysed the risk tolerance of an investor with respect to the expected utility of the monthly returns. The degree of risk aversion or risk seeking was computed separately for each month. Now, we extend the time period of the examination. Which degree of risk tolerance does an investor need to opt for Premium Bonds if the investment period comprises a longer time horizon? Are there any significant changes in comparison with our previous results? We look at an individual who intends to invest a lump sum at a particular point of time for either five, ten or twenty years. The first choice is to buy a risk-free bond with a fixed interest rate depending on the current interest rate level. There are no coupon interest payments during the investment period (zero coupon bond). Hence, the investor collects all the interest and compounded interest at maturity. The CRRA utility is calculated from this final payment. To simplify matters, we only study the case of a non-taxpayer. As a reference for these calculations, we use the Bank of England UK yield curve (Bank of England, 2006) based on UK government bonds (gilts). Employing this data, we identify the nominal spot rates for investments with investment periods between one month and 25 years. Since the yield curve records start in this year, we begin with January, 1979. We assume that an individual invests £1 at the beginning of January, 1979. Then we calculate the risk coefficients for three time horizons: twenty years with maturity at the beginning of January, 1999, ten years with maturity in January, 1989, and five years with maturity in January, 1984. At the end of the maturity, the investor gets his principal refunded. For the calculation of the interest payment, we use the monthly discrete interest rates calculated from the compounding interest rates of the spot curves.

For the Premium Bond, we construct the following investment strategy. The investor buys one bond worth £1 at the end of December 1978. This means that he/she will participate in the prize draw for the first time at the beginning of February 1979. Now he/she either wins a prize or not. If he or she wins, we assume that the prize is invested at the current spot rate exactly for the remaining time period till the beginning of 1999, 1989 or 1984. Then, for each prize in each draw, we calculate the value including interest at maturity. For that purpose, we need the exact monthly spot rates. Due to the fact that the published yield curves usually only provide spot rates semi-annually, we estimate the required monthly spot rates by non-linear interpolation. We therefore apply the Svensson-Nelson-Siegel approach (Nelson-Siegel, 1987 and Svensson, 1994). By iteration, it is possible to infer from any yield curve the parameters of the Svensson-Nelson-Siegel equation. For the twenty year time period, we calculate the parameters of 240 equations. With this model, it is possible to estimate quite precisely the spot rate for any maturity. With these "invested" prizes, we now compute for each month the expected CRRA utility of the Premium Bond at maturity. For consistency reasons, the principal is refunded together with the last prize draw. In order to obtain one single indifference risk coefficient, we use the same α in all Premium Bond utility functions and in the utility function of the risk-free investment. The indifference value of α is determined by iteratively finding the value where the sum of all Premium Bond utilities and the utility of the risk-free spot rate investment becomes equal. The results are presented in table 11.

Place table 11 here

A further test with £1000 wealth and £100 invested results -0,178989 for the twenty year maturity. Why does the 20 year investment require more risk loving than the ten or five year investments? The answer is that it depends on the current interest level at the time of the investment decision. In 1979, the overall interest level was quite high, which favoured a long-term fixed investment. Due to the fact that our Premium Bond investment strategy always depended on the current spot rate each month, the decreasing interest level in the following years had a negative effect. This is the reason why the 5 years investment shows an indifference risk coefficient of only -0.12829. In the first 5 years, the interest level was much higher than in the remaining time. Figure 6 illustrates how the monthly spot rates with maturity in January, 1999 changed over the 20 year period.

Place figure 6 here

Deciding in favour of the Premium Bond in 1979 and then investing potential winnings at the current spot rate with maturity in January, 1999, had three potential risks. The first is the odds of winning a prize at all, the second is that the nominal interest rate of the Premium Bond may be unfavourably changed and the third risk is that the spot rates for the investment of the prizes are also subject to changes. However, taking into account these three factors, the required degree of risk seeking seems relatively quite low again.

2.5 Comparison with the Approximation Formula of Pratt (1964)

In some empirical studies on individual risk preferences, a popular approximation is used in order to calculate the risk coefficients. Pratt's formula developed in 1964, does not comprise wealth, meaning that it is only an approximation for the constant absolute risk aversion. The approximation is calculated as:

$$\alpha = -\frac{u''(x)}{u'(x)} = \frac{\sum_{k=1}^{K} f_k Z_K - \lambda}{\frac{1}{2}\lambda^2 + \frac{1}{2}\sum_{k=1}^{K} f_k Z_k^2 - \lambda \sum_{k=1}^{K} f_k Z_k} \qquad (4)$$

f stands for the probability of winning the prize Z and λ expresses the price for participating in the lottery. The results of our iteratively computed risk coefficients are a good opportunity to check if these two ways of calculating risk measures provide the same results or if there are some biases. We perform the comparison on the basis of our first myopic analysis (Premium Bond vs. BoE base rate), since this provides the longest time period (see table 12).

Place table 12 here

Testing consistency, we divide Pratt's approximated Alphas by the iteratively determined risk coefficients, showing that the approximation overestimates the indifference risk coefficients determined by equating. Only for the higher income tax brackets do the approximations lead to an underestimation. In general, the lower the tax rate the higher the degree of overestimation. Table 13 depicts the average deviations.

Place table 13 here

Pratt's approximation and our method produce quite different values for the Premium Bond sample.

2.6 Conclusions from the Risk Tolerance Analysis

After this analysis, we can draw some conclusions on risk preferences as a requisite for the purchase of Premium Bonds. The first noteworthy finding is that the individual tax rate is a crucial factor in the investment decision. Also, the Premium Bond as a lottery bond can still appeal to risk-averse investors, and, being tax-free, it is even more appealing for high-income taxpayers. Basic rate taxpayers require a small degree of risk seeking. The less taxes an individual pays, the more risk loving he or she needs to be. However, in comparison to other empirical studies, the indifference risk coefficients, especially in terms of risk seeking are moderate, which is consistent with the Premium Bond's features: no risk of losing the principal and the stake is only the foregone interest of a certain investment. Due to their large proportion of the population, basic rate taxpayers seem to be the main contributors to the success of the Premium Bond,. As our analysis reveals, the indifference level of risk seeking, especially for this tax bracket, has decreased in the last years. Furthermore, due to recently stable margins of interest, the volatility of the risk coefficients has decreased, and that additionally limited the risk. Consequently, the individual level of risk tolerance theoretically allows the purchase of Premium Bonds for more investors. On the other hand, since the indifference risk coefficients for higher rate taxpayers tended toward risk neutrality, it is reasonable to assume that for this tax class the Premium Bonds became somewhat less attractive. However, these investors still enjoy the best of both worlds. The remaining taxpayers (starting and basic tax brackets) definitely have to exhibit the highest degrees of risk seeking. This fact, coupled with mostly limited funds, explains why these individuals predominantly hold small numbers of bonds only for gambling rather than for investment purposes. Our conclusions thus suggest why so many Britons invest in Premium Bonds. While the overall risk, measured by expected utility theory, is relatively small, investors still get a thrill from gambling. But, it is not yet proven that sales and risk measures are actually correlated. We will shed light on this question in the next section.

3. Factors Influencing Success

Our objective now is to identify the main factors that explain the development of sales over time. We also gauge the potential influence of the indifference risk coefficients treated previously. Therefore, we run several statistical tests and conduct a regression analysis. The basis for our procedure is data on sales, repayments and net contributions from October, 1969 to April, 2006.

3.1 Identification

In a recent study, Campbell (2006), finds that due to complex household finance problems many make investment errors. Especially less educated households frequently make significant mistakes. Among other things, they often miss investing in certain financial products due to ignorance. Since the Premium Bond design is straightforward, it appeals to virtually every household. The relatively low average amount invested (£1,400) along with the impressive number of about 23 million bond holders reflect that. Therefore, the simple design of the Premium Bond could be one reason for its remarkable success.

Place figure 7 here

Raising the first prize from £250,000 to £1,000,000, the largest increase of the jackpot ever, caused the first boom in sales in April, 1994. The second jump in sales in May, 2003, also the most voluminous one until the present, can be attributed to the increase of the maximum holding from £20,000 to £30,000. Apparently, many people grabbed the chance to place additional funds in Premium Bonds. Finally, the most recent peak in sales was in August, 2005, when NS&I introduced a second million pounds as a jackpot. At first glance, it seems that the value of the first prize and maximum holding are important factors. To what extent, however, do they interfere with sales and are there any significant additional factors? We need to point out here that special events, like increasing the jackpot, are usually accompanied by advertising campaigns encouraging investors to buy the bond. However, as this kind of data is not available, it is impossible at the moment to take into account advertising expenditures and their effectiveness as a driving factor for the Premium Bond's success.

Glancing at repayments, we find the ratio between repayments and sales for the full period averages 53.4%, meaning that each month on average 53.4% of the sales were refunded to the investors. A straightforward trend analysis reveals this ratio recently declined. From the sales peak in 1994 until April, 2006, the average repayment ratio added up to only 43.7%. This is just one reason why the net contribution also increased substantially since 1994. Figure 8 shows how the total stock of the Premium Bond then grew much faster.

Place figure 8 here

We also want to discover whether sales were affected by CARA or CRRA coefficients changing over time. We take the time series of four indifference risk coefficients to clarify whether they have a short-run or long-run influence on sales. Since the basic rate taxpayers represent the largest group, we mainly use the risk coefficients of this tax class. The other coefficients follow roughly the same pattern, gleaning rather similar results. To identify causal correlations, we employ Granger Causality Tests (Granger, 1969), allowing us to test whether, after controlling for past Y (e.g. sales), past changes of X (e.g. CRRA) help to forecast Y (Wooldrigde, 2003). One of Granger's crucial assumptions for testing causality is that the variables do not follow a distinct trend, implying they must be stationary. Because working with non-stationary variables can lead to spurious regressions and inferences, we first perform an Augmented Dickey-Fuller Test (Dickey and Fuller, 1979) to discover if the data have unit roots attesting non-stationarity. As the graphs suggest, the variable "Sales" is integrated of order 1. The t-statistic of the test amounts to 1.325642 and the p-value to 0.9988[2] which proves the presence of one unit root. Conducting this test on all risk coefficient time series shows that these variables are stationary. For consistency reasons, we still use the first differences since the variable "Sales" is non-stationary. This procedure is a common way of dealing with a non-stationary time series. If there is only one unit root in a variable, differencing once generates a stationary time series. However, by doing that, we can only observe the changes in the variables and we lose information included in the levels.

[2] MacKinnon (1996) one-sided p-values.

The Granger Causality Test works like this. First, we test the null hypothesis that changes in the risk coefficient (e.g. CRRA) do not cause changes in sales. Therefore, we use an autoregressive model:

Unrestricted regression: $\Delta sales_t = \eta_0 + \beta_1 \Delta sales_{t-1} + \beta_2 \Delta CRRA_{t-1} + \varepsilon_t$ (5)

Restricted regression: $\Delta sales_t = \eta_0 + \beta_1 \Delta sales_{t-1} + \varepsilon_t$ (6)

The first regression expresses that changes in sales depend on changes in sales in t-1 and on changes in CRRA in t-1. The error term has an expected value of zero. The second regression is for the significance test. In this equation, the influence of CRRA is set to zero. To make sure that there only exists a unidirectional causality, we also test if changes in sales cause changes in CRRA. Thus,

Unrestricted regression: $\Delta CRRA_t = \eta_0 + \beta_1 \Delta CRRA_{t-1} + \beta_2 \Delta sales_{t-1} + \varepsilon_t$ (7)

Restricted regression: $\Delta CRRA_t = \eta_0 + \beta_1 \Delta CRRA_{t-1} + \varepsilon_t$ (8)

Table 14 reports the results of the Granger Tests on the risk coefficients. We test 1, 3 and 6 lagged periods.

Place table 14 here

According to the tests, past changes in the risk coefficients do not help to forecast sales. Further tests with different time horizons yield negative results as well. We thereby conclude that there are no short-run effects from changes of the indifference risk coefficients on sales. However, it is still theoretically possible that sales and the risk coefficients exhibit a long-run correlation. One might assume that steadily increasing risk measures (higher positive values) cause increases in sales as more and more risk averse investors find Premium Bonds attractive. A long term equilibrium emerges if two non-stationary variables X and Y are cointegrated (Engle and Granger, 1987). This means that there is a cointegration parameter d such that $C = Y - dX$ is stationary. The relationship $Y = dX$ describes a long-run relationship. In our case, the explanatory variables are stationary, but the explained variable "Sales" follows a distinct trend. Therefore, the precondition of a cointegrating relationship is not fulfilled and thus we cannot identify a long-run equilibrium. Because of this, we cannot prove that risk coefficients are directly accountable for the impressive sales recently. We also test for some potential bond related and macroeconomic factors that were partially mentioned in discussions with experts of NS&I.

Place table 15 here

However, as table 15 shows, these factors do not produce robust results. The Johansen Test (Johansen, 1991, 1995) indicates no cointegration and the Granger Test also states no clear causal relationships. Over time, the sign of the correlation between the FTSE100 and sales of the Premium Bond frequently changed. Between 1984 and 2000, there was a positive correlation between sales and the FTSE100. However, after 2000, while the stock market declined, sales of the Premium Bond increased. This negative correlation flipped back again in 2003. Hence, Granger Tests on this variable do not produce distinct results. With respect to the absolute interest rate of the Premium Bond, the usability is also very limited. Of course, high interest rates should attract many investors: however, in the past 20 years, the overall interest level gradually decreased and therefore the nominal interest rate fails to account for Premium Bond sales. Furthermore, Granger Tests on the ratio between the interest rate of the Premium Bond and the Income Bond also do not indicate a direct relationship. In the case of the Premium Bond, we cannot prove the frequently discussed importance of prize distribution variance (Walker and Young, 2001). However, the third moment of the prize distribution is often considered crucial. Therefore, we test in particular the prize skewness as a factor favouring the decision to purchase Premium Bonds. In the first prize draw in July 1957, NS&I gave away prizes between £25 (1,920 times) and £1,000 (96 times). In the last fifty years, the distribution of prizes has been adjusted from time to time resulting in a change of the prize skewness. For example, NS&I raffled 1,200,611 times £50 and 2 times £1m in the prize draw in November 2006. This design of the prize distribution follows what behavioural theory stipulates: a lottery should offer a large number of small prizes to reduce holder's fatigue from the low likelihood of winning. On the other hand, it should also offer a small number of large prizes (creating skewness) to make the lottery attractive and to keep the thrill (Shapira and Venezia, 1992). This theory works well because people generally overestimate the very low probability of winning a high prize (Camerer and Kunreuther, 1989). The following figure shows the skewness of each prize draw along with the sales from October, 1969, to April, 2006.

Place figure 9 here

According to figure 9, it seems that skewness is indeed positively correlated with sales. However, in August, 2005, we observe evidence contradicting this theory. Introducing the second £1m prize reduced skewness but simultaneously led to higher sales. We assume that, besides advertising efforts, the prospect of winning such a large sum always attracts people, although skewness is reduced. As we have noted, people generally overestimate low probabilities, so the doubling of the chance is also overestimated. It is also interesting that apparently investors do not consider the purchasing power of the prizes. The nominal £1m of August 2006 was worth £1,38m expressed in money in April, 1994. This means that the actual purchasing power of the prize declined substantially since 1994. One may think that this would make the first prize less attractive, but sales figures prove the opposite.

Next, we perform some tests to describe the relationship between skewness and sales. In order to test for causality, we apply the Granger Test once more. As already mentioned, the variables must not have a unit root. The Augmented Dickey-Fuller Test on the variable "Skewness" produces a t-statistic of 0.070854 and a p-value of 0.9632. Again, there is strong statistical evidence for one unit root.[3] We rule out the presence of a second unit root by

[3] The Dickey-Fuller Test can be biased in the presence of structural breaks. Splitting the time frame in two pieces before and after the break in 1994 also confirms unit roots.

repeating the tests with 1st differences. In order to avoid spurious inferences, we continue with differenced variables. Table 16 reports the results of the Granger Test with several lag lengths.

Place table 16 here

According to these test results, one can say that changes in skewness do not cause changes in sales. As mentioned before, this is a short-term analysis without consideration of level information. It seems reasonable that a small change in the distribution of prizes, only causing marginal changes in skewness, has no direct effect on sales. Only big jumps, e.g. due to the introduction of a very high prize directly cause higher sales. It seems that the overall level is more important than the discrete changes. The graph suggests this long-term relationship. To confirm this, we perform the Johansen Cointegration Test.

The test states that there is one cointegrating relationship between sales and skewness at the 1% significance level, underlining the theory that skewness is correlated with sales. Further analyses on skewness will follow in section 3.2.

Place table 17 here

As we remember (figure 7), one of the peaks in sales came from an increase of the maximum holding from £20,000 to £30,000. Is there also a statistical relationship? The unit root test produces a t-statistic of -0.216550 and p-value of 0.9335. Again, this variable contains a unit root in the time between October, 1969, and April, 2006. As we have seen, in this case, Granger Causality Tests would only be valid with first differences. However, increases in the maximum holding occurred very rarely, making Granger Causality Tests over the full-time period rather ineffective. At least, the peak in sales in May 2003 can be proven. In the period between December, 2002 and October, 2004 the Granger Causality Test produces a F-statistic of 3.40897 (p-value: 0.04247) at 6 lags and 3.08514 at 7 lags, indicating a unidirectional causality. It is even more interesting if there is some kind of long-term relationship as well.

We assume that a high maximum holding gives incentives for wealthy higher rate taxpayers to invest up to the maximum holding and for basic rate taxpayers to increase their personal holding according to their possibilities. Therefore, we test once again for cointegration. The Johansen Cointegration Test does not find significant cointegrating relationships over the entire time period. However, from January, 1980, to April, 2006, and also after the structural break in 1994, we find evidence for cointegration.

Place table 18 here

Table 18 shows that, since 1980, the level of the maximum holding is also a factor significantly related to sales in the long-run. This means that higher amounts of the maximum holding cause increasing sales in the long run.

3.2 Regression Analysis

In the following section, we construct a regression model building on previous results. The dependent variable is "Sales". Since net contribution depends on repayments which in turn depend on a variety of factors, the sales time series best fits our purpose. Our previous analysis supports the assumption that skewness of the prizes denoted "Skewness" and the maximum holding denoted "MaxInvest" are some of the key influencing factors. As the traditional risk coefficients proved inessential, we excluded them. The other previously tested factors also do not seem to have decisive influence and, furthermore, as tests show, do not improve the quality of the regression model. Consequently, we restrict our observations to the two most salient influencing factors. Thus, the independent variables of the regression are "Skewness", "MaxInvest", and a correction variable which we will explain below. Since we do not expect "Skewness" and "MaxInvest" to be influenced by the sales, we treat these variables as exogenous and do not apply multivariate models such as a vector autoregressive regression. Due to multicollinearity, we exclude a manifest factor like the value of the first prize. Since skewness is calculated from this figure, the regression would be biased. More important is the variable “number of jackpots”. As mentioned before, we find a contradiction to the theory that only increasing skewness causes additional sales. Although investors apparently actually prefer “skewed” prizes, this general statement proves to be incorrect for changes in the number of the jackpots. As we can detect in the sales time series, the increased chance to win a very high prize attracts people even though the skewness deteriorates. In order to include this phenomenon, we use a dummy variable denoted “MPDummy”, which is equal to zero as long as there is only one first prize and changes to one in July, 2005. The dummy variable alleviates the problem of the lower value of the skewness and consequently helps to obtain better forecasts after August, 2005. Sales of the Premium Bond, as we have seen, have been relatively steady until the end of 1993. When NS&I introduced the £1m, the investment behaviour of the investors considerably changed. Consequently, the parameters of the model changed as well. A regression over the full period would be biased. We thereby focus our parameter estimation from October, 1993, to April, 2006, which comprises 151 observations starting with OLS regressions.

Place table 19 here

As “Sales”, “Skewness” and “MaxInvest” are non-stationary variables, we need to exercise caution when performing regressions. The Durbin-Watson statistic of 1.13345 indicates the presence of serial correlation. If the residuals of a regression model are correlated, the coefficient estimates are not BLUE (Best Linear Unbiased Estimator). For verification, we perform a Breusch-Godfrey Serial Correlation LM Test, allowing examination of the relationship between the residuals and several of its lagged values at the same time. We take into account 10 lagged periods. The F-statistic of 7.642053 (p-value = 0.0000) and Obs*R-squared of 53.38911 (p-value = 0.0000) prove the existence of strong autocorrelation in the residuals. The Bera-Jarque statistic amounts to 1445.603 (p-value 0.0000). According to the test, normal distribution of the residuals can be rejected. In our case, we bypass this fact. If a sample is sufficiently large, the violation of normality is virtually inconsequential (Brooks, 2004). Since our sample contains more than 150 observations, we appeal to the central limit theorem stating that test statistics will asymptotically follow the appropriate distributions even in the absence of error normality. Heteroscedasticity is another frequently encountered problem while constructing a regression model. Since serial correlation invalidates any test for heteroscedasticity, we first have to correct this. One solution when autocorrelation is present is switching from a static to a dynamic model. This type of model does not only take into account contemporaneous relationships, but also contains previous values of the explanatory and/or explained variables. In order to control serial correlations, we use autoregressive processes (AR) in the regression model. An autoregressive model of order p is denoted by AR(p) expressed by equation (9):

$$y_t = \eta + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t \quad (9)$$

However, this autoregressive model is only applicable to stationary variables. As before, we proceed with first differences of all variables. A second test for unit roots confirms that now all variables are stationary. According to the Akaike and Schwarz information criteria, an AR(2) model fits best. After this correction, the Durbin-Watson statistic reports 2.002 and the Breusch-Godfrey Test a F-statistic of 1.047078 (p-value 0.407811) at 10 lagged periods. Serial correlation no longer presents a problem. The White Heteroscedasticity Test produces a F-statistic of 2.6315 (p-value 0.036683), meaning we assume that the variances of the regression residuals are not homogeneous. Furthermore, the ARCH LM Test indicates autoregressive conditional heteroscedasticity. In order to control this, we tried several ARCH, GARCH and EGARCH models. However, although we were able to reduce the variances of the coefficient estimates, we faced significance problems with the variables and the variance coefficient estimates. Since forecast analyses showed that the goodness of the GARCH models is not superior, it was reasonable to keep to the OLS method. The Newey-West method for heteroscedasticity consistent errors and covariance helps to minimize the problem of heteroscedasticity. Table 20 presents our model.

Place table 20 here

The above regression model then leads to an autoregressive integrated moving average model ARIMA(2,1,0). That means we use an autoregressive process of order 2, the variables are differenced one time, and we do not include any moving average processes as the model becomes instable and forecasts only marginally improve. Although the model looks fairly simple, according to adjusted R-squared, it can still explain 49.1% of the variance. The Ramsey RESET test cannot prove misspecification (F-statistic 2.690756). In-sample forecast tests should help us uncover how well the model works. Since we need all the observations for an appropriate parameter estimation, we cannot perform out-of-sample forecast tests. There are two different kinds of in-sample forecasts: a static and a dynamic forecast. The static forecast is a sequence of one-step ahead forecasts. For each estimation, the actual value of the lagged dependent variable is used for the autoregressive term. In the dynamic procedure, the forecasted lagged dependent variables determine the current forecast. The estimations thus become inaccurate the longer the forecasting sample. In our case, we choose the period December, 2001, to April, 2006, which comprises 53 observations. Figure 10 shows the series of the static forecast.

Place figure 10 here

According to the forecast summary statistic, our ARIMA model provides fairly good one-step ahead forecasts of sales. Let us see what the dynamic forecast looks like for the same period as before. Note that all variables in the model are differenced once. Figure 11 compares the actual and the forecasted values.

Place figure 11 here

While the model can depict the jump in sales in May, 2003, to a certain degree, it does overestimate sales in 2005 and after. This differenced dummy variable does not provide a good fit. Apparently, the introduction of the second jackpot changes the parameters of the regression model, meaning the shock stays in the systems. Therefore, we retest with a non-differenced dummy variable which maintains value 1 after July 2005.

Place figure 12 here

As figure 12 illustrates, the forecast now fits much better for the time after the introduction of the second jackpot. Generally, from the graphs above, we conclude that while the static forecast works quite well, the dynamic forecast provides only a rough estimation. The error with dynamic forecast increases because of the lack of updated actual information on the dependent variable. Yet, our parsimonious model is indeed capable of predicting sales to a certain degree, applying especially to one-step ahead forecasts. The autoregressive process brings level information of the sales into the model, considerably increasing the accuracy. We have shown that “Sales” is cointegrated with “Skewness” and also with “MaxInvest”. It is thus reasonable to assume that the autoregressive process also represents the level information from the long-run relationships previously lost by differencing. A shortcoming of this regression model is that it fails to explain short-run volatility. Our model captures salient factors and helps to explain additional sales. The regression model statistically confirms two statements. First, especially major changes in the skewness of prizes and maximum holding cause boosts in sales and, second, past sales which are cointegrated with these factors also give useful indications for future sales.

4. Conclusion

The objective of this paper is to conduct an empirical analysis of the British Premium Bond. What prompts so many investors to buy and hold a lottery bond with overall risky payouts? In the first step, we calculate the degree of risk tolerance an investor needs to exhibit in order to be indifferent between the Premium Bond and a risk-free investment. A central issue is the discrimination of the different tax classes. Premium Bond prizes are tax-free, making them more or less attractive for certain taxpayers. Basically, we find that the indifference risk coefficients are surprisingly close to risk neutrality, especially in recent years. Since 1993, the Premium Bond has been managed very carefully. In general, the indifference risk levels are now relatively stable. For high income taxpayers, even a low degree of risk aversion is sufficient to invest in Premium Bonds. However, for those, the bond became somewhat less attractive in terms of required risk tolerance in the past thirty years. The basic rate taxpayers who seem to be the main contributors only have to exhibit a slight degree of risk seeking behaviour. The two remaining taxpayer groups do not profit from the tax-relief to the same extent and therefore require a more risk seeking preference. Basically, the Premium Bond turns out to be not especially risky using conventional measures. A long-run analysis confirmed that. There is no perceived threat of losing the principal and the fees for participating are just the foregone interest payments of a certain alternative investment. All these features make the Premium Bond attractive for a broad spectrum of investors.

We also wanted to explore the remarkable success of the Premium Bond. To prove that prize skewness and the maximum holding significantly influence sales, we conduct Granger Causality Tests and Johansen Cointegration Tests. The result is that short-run effects are only distinctive when major changes occur, such as the introduction of a new first prize, but there is evidence of long-run relationships. An ARIMA model confirms the influence of skewness and maximum holding on sales. Interestingly, CARA- and CRRA-risk coefficients do not statistically affect sales. Apparently, classical risk preferences do not play a major role in this investment decision.

References

Arrow, K. “Aspects of the Theory of Risk-bearing” Helsinki: Yrjö Jahnssonin Säätiö, 1965.

Adam, S., and J. Browne. “A Survey on the UK Tax System” Institute of Fiscal Studies, Briefing Note No. 9, 2006.

Beetsma, R., and P. Schotman. “Measuring Risk Attitude in a Natural Experiment: Data from the Television Game Show LINGO” The Economic Journal 111, 2001, 821-848.

Bhattacharyya, N., and T. A. Garrett. "Why People Choose Negative Expected Return Assets - An Empirical Examination of a Utility Theoretic Explanation" Federal Reserve Bank of St. Louis, Working Papers 2006-014, March 2006.

Brooks, C. “Introductory Economics in Finance” Fifth reprint, Cambridge, 2004.

Camerer, C. F., and H. Kunreuther. “Decision processes for low probability events: policy implications” Journal of Policy Analysis and Management 8(4), 1989, 565-592.

Campbell, J. Y. “Household Finance" Presidential Address to the American Finance Association, Journal of Finance 61, August 2006, 1553-1604.

Dickey, D.A., and W.A. Fuller. ”Distribution of Estimators for Time Series Regressions with a Unit Root” Journal of American Statistical Association 74, 1979, 427-431.

Donkers, B., B. Melenberg, and A. Van Soest. “Estimating Risk Attitude using Lotteries: A Large Sample Approach” The Journal of Risk and Uncertainty 22, 2001, 165-165.

Engle, R. F., and C. W. J. Granger. “Co-integration and Error Correction: Representation, Estimation, and Testing” Econometrica 55, 1987, 251–276.

Florentsen B., and K. Rydqvist. ”Ex-day behavior when investors and professional traders assume reverse roles: The case of Danish lottery bonds” Journal of Financial Intermediation 11, 2002, 152—175.

Garrett, T. A., and R. S. Sobel. “Gamblers Favor Skewness, not Risk: Further Evidence from United States’ Lottery Games” Economics Letters 63(1), 1999, 85–90.

Golec, J., and M. Tamarkin. “Bettors Love Skewness, Not Risk, at the Horse Track” Journal of Political Economy 106-1, 1998, 205–225.

Granger, C. W. J. “Investigating Causal Relations by Econometric Models and Cross-Spectral Methods” Econometrica 37, 1969, 424-438.

Granger, C. W. J., and P. Newbold. “Spurious Regressions in Econometrics” Journal of Econometrics 2, 1974, 111–120.

Green, R. C., and K. Rydqvist. ”The Valuation of Non-Systematic Risks and the Pricing of Swedish Lottery Bonds” Review of Financial Studies 10, 1997, 447-480.

Green, R. C., and K. Rydqvist. “Ex-day behavior with dividend preference and limitations to short-term arbitrage: the case of Swedish lottery bonds” Journal of Financial Economics 53, 1999, 145—187.

Guillén, M., and A. E. Tschoegl. “Banking on Gambling: Lottery-Linked Deposit Accounts” Journal of Financial Services Research 23 (3), 2002, 219-231.

Johansen, S. “Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models” Econometrica 59, 1991, 1551–1580.

Johansen, S. “Likelihood-based Inference in Cointegrated Vector Autoregressive Models” Oxford: Oxford University Press, 1995.

Jullien, B., and B. Salanié. “Estimating preferences under risk: The case of racetrack bettors” Journal of Political Economy 108, 2000, 503-530.

MacKinnon, J. G. “Numerical Distribution Functions for Unit Root and Cointegration Tests” Journal of Applied Econometrics 11, 1996, 601-618.

MacKinnon, J. G., A. A. Haug, and L. Michelis. “Numerical Distribution Functions of Likelihood Ratio Tests For Cointegration” Journal of Applied Econometrics 14, 1999, 563-577.

National Savings and Investments. “Product Accounts 2003/04”, 2004.

National Savings and Investments. “Product Accounts 2002/03”, 2003.

National Savings and Investments. “Product Accounts 2001/02”, 2002.

National Savings and Investments. “Product Accounts 2000/01”, 2001.

National Savings and Investments. “Annual Report and Accounts 2004/05”, 2005.

National Savings and Investments. “Premium Bond Facts and Figures”, 2006.

National Statistics. “Retail Prices Index: monthly index numbers of retail prices 1948-2006”, 2006, URL: http://www.statistics.gov.uk/STATBASE/tsdataset.asp?vlnk=229, downloaded: 06.10.2006

Nelson, C. R., and A. F. Siegel. “Parsimonious Modeling of Yield Curves”, Journal of Business 60, 1987, 473-489.

Pfiffelmann, M. “Which optimal design for LLDAs?” Working Paper, September 2006, URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=931130, downloaded: 10.10.2006

Pratt, J.W. "Risk Aversion in the Small and in the Large" Econometrica Vol. 32, 1964, 122-36.

Shapira, Z., and Y. Venezia. “Size and frequency of prizes as determinants of the demand for lotteries” Organizational Behavior and Human Decision Processes 52, 1992, pp. 307-318.

Svensson, L. E. O. “Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994” IMF Working Paper, International Monetary Fund, 1994.

Ukhov, A. “Preferences Toward Risk and Asset Prices: Evidence from Russian Lottery Bonds” Working Paper, November 2005, URL: http://ssrn.com/abstract=841425, downloaded: 10.10.2006

Walker, I., and J. Young. “An Economist's Guide to Lottery Design” Economic Journal, 2001, vol. 111, issue 475, 2001, F700-722.

Wooldridge, J. M. “Introductory Econometrics: A modern approach” Second edition, Manson, 2003.

Tables

Table 1 Total funds invested by product (£ billion) (31 March). Since 2001 Premium Bonds became more and more important for NS&I and in March 2006 accounted for 42.37% of the total funds invested. In 2001 most funds were invested in Saving Certificates. However, in 2002 the Premium Bond became the product with the largest share. Remarkably, according to NS&I's annual report from 2004 to 2005, virtually the complete growth in funds invested is attributed to sales of Premium Bonds. Sources: NS&I Product Accounts 2001-2004, NS&I Annual Report 2005, NS&I Web Page

	2001	**2002**	**2003**	**2004**	**2005**	**2006**
NS&I Total	62.7	62.3	63.1	66.5	68.5	73.4
Premium Bond	15.4	17.3	19.7	24.3	26.6	31.1
PB percentage of total	24.56%	27.77%	31.22%	36.54%	38.83%	42.37%

Table 2 Number and value of prizes awarded in November 2006. Source: NS&I (2006) Media Centre

	Lower Value 87%		**Medium Value 6%**		**Higher Value 7% of prize fund**					
Prize £	**50**	**100**	**500**	**1,000**	**5,000**	**10,000**	**25,000**	**50,000**	**100,000**	**1,000,000**
Number	1,200,611	138,030	6,111	2,037	157	78	31	16	8	2

Prize value	**Number of prizes**	**Interest Rate p.a.**
£ 84,866,050	**1,347,081**	**3.15%**

Table 3 Income taxation in the United Kingdom 2006-07.
Source: Adam and Browne (2006)

	Tax rate	**Band**
Higher Rate	40%	over £33,300
Basic Rate	20%	£2,151 - £33,300
Starting Rate	10%	0 - £2,150

Table 4 Risk coefficients: Premium Bond vs. BoE Rate.

Alpha	Mean	Median	StdDev	Maximum	Minimum	Values
CARA (no tax)	-0.00009446	-0.00001961	0.00028220	0.00115522	-0.00070252	592
CARA (starting tax)	0.00014644	-0.00000433	0.00091672	0.00801365	-0.00070252	388
CARA (basic tax)	0.00097530	-0.00000362	0.00283739	0.01614532	-0.00070252	592
CARA (higher tax)	0.00949976	0.00420088	0.01251360	0.08657697	-0.00002123	592
CRRA (no tax)	-0.04027261	-0.03900994	0.02598947	0.01190405	-0.10862350	592
CRRA (starting tax)	-0.01680483	-0.01888015	0.02045698	0.03153098	-0.06894967	388
CRRA (basic tax)	-0.00792707	-0.00875098	0.02468149	0.05032705	-0.07120177	592
CRRA (higher tax)	0.03263185	0.03129072	0.02834992	0.13427937	-0.03344846	592

Table 5 Risk coefficients: Premium Bond vs. Income Bond.

Alpha	Mean	Median	StdDev	Maximum	Minimum	Values
CARA (no tax)	-0,00001114	-0,00000468	0,00000755	-0,00000253	-0,00002249	291
CARA (starting tax)	0,00001654	-0,00000366	0,00019066	0,00180659	-0,00001539	177
CARA (basic tax)	0,00002682	-0,00000302	0,00032827	0,00396062	-0,00001914	291
CARA (higher tax)	0,00333332	0,00249892	0,00300179	0,01323053	-0,00001486	291
CRRA (no tax)	-0,03836096	-0,03428748	0,01488229	-0,00662202	-0,07845371	291
CRRA (starting tax)	-0,01373163	-0,01589249	0,01030152	0,01567519	-0,03212587	177
CRRA (basic tax)	-0,00985924	-0,00798917	0,01275165	0,02197919	-0,04677832	291
CRRA (higher tax)	0,02021929	0,02167165	0,01710026	0,05732383	-0,02308127	291

Table 6 Risk coefficients: Premium Bond vs.BoE rate with wealth and £1,400 invested.

Alpha	Mean	Median	StdDev	Maximum	Minimum	Values
CRRA (no tax)	-0.12950602	-0.13250552	0.07524556	0.05827165	-0.31353273	592
CRRA (starting tax)	-0.06055116	-0.07599001	0.07831720	0.14609413	-0.19850094	388
CRRA (basic tax)	-0.01932498	-0.05932223	0.15324314	0.49433104	-0.29352230	592
CRRA (higher tax)	0.49880718	0.35200058	0.54017149	3.59850239	-0.20027504	592

Table 7 Risk coefficients with wealth vs. without wealth (Premium Bond vs. BoE rate). The indifference risk coefficient of the CRRA utility function is on average 12.36 (higher tax rate) times higher if wealth and a £1,400 investment is taken into the calculation.

With wealth	Average multiplier
CRRA (no tax)	2.70
CRRA (starting tax)	3.91
CRRA (basic tax)	6.03
CRRA (higher tax)	12.36

Table 8 Maximum Holding in Premium Bonds.

Time	£ Maximum Holding
06/1957 - 03/1964	500
04/1964 - 03/1980	1,000
04/1980 - 03/1993	10,000
04/1993 - 04/2003	20,000
Since 05/2003	30,000

Table 9 Risk coefficients: Premium Bond vs. BoE rate with wealth and maximum investment.

Alpha	Mean	Median	StdDev	Maximum	Minimum	Values
CRRA (no tax)	-0.0693620	-0.0639166	0.0455772	0.0276244	-0.2008728	592
CRRA (starting tax)	-0.0354924	-0.0433263	0.0462911	0.0729853	-0.1469507	388
CRRA (basic tax)	-0.0156025	-0.0282268	0.0795302	0.2591578	-0.2123467	592
CRRA (higher tax)	0.1798794	0.1198804	0.2201104	1.5909653	-0.1053037	592

Table 10 Risk coefficients: Premium Bond vs. Income Bond with wealth and £1,400 invested.

Alpha	Mean	Median	StdDev	Maximum	Minimum	Values
CRRA (no tax)	-0.1013200	-0.0961102	0.0330789	-0.0197167	-0.1901446	291
CRRA (starting tax)	-0.0527910	-0.0645525	0.0401676	0.0665196	-0.1217610	177
CRRA (basic tax)	-0.0493301	-0.0429346	0.0620363	0.1650962	-0.2089394	291
CRRA (higher tax)	0.2277819	0.2128044	0.1810590	0.7260126	-0.1525437	291

Table 11 Risk coefficients: Premium Bond vs. Spot rate investment (non-taxpayer). Like in the previous myopic cases a non-taxpayer needed to be risk loving. The indifference risk coefficient of the 20 years investment period is more negative than the ones with 10 and 5 years maturity.

	Alpha
CRRA 20 years	-0.15719470
CRRA 10 years	-0.12815369
CRRA 5 years	-0.12829244

Table 12 Risk coefficients: Alpha Approximation, Premium Bond vs. Income Bond. In general the results indicate the same conclusions. Non- and starting rate taxpayers need to be risk loving, basic rate taxpayers are close to risk neutrality and for higher rate taxpayers a sufficient degree of risk aversion is enough to decide in favour of the Premium Bonds. However, the Pratt values of Alpha differ clearly form the iteratively calculated values.

Alpha	Mean	Median	StdDev	Maximum	Minimum	Values
Alpha (no tax)	-0.00047306	-0.00011492	0.00079735	0.00056373	-0.00394738	592
Alpha (starting tax)	-0.00029916	-0.00003073	0.00073421	0.00101467	-0.00302633	388
Alpha (basic tax)	0.00008481	-0.00001428	0.00052638	0.00191654	-0.00121604	592
Alpha (higher tax)	0.00049658	0.00007466	0.00080411	0.00281841	-0.00008024	592

Table 13 Comparison: Alpha Approximation vs. iteratively determined risk coefficients. The average multiplier expresses the factor by which the estimated risk coefficient is larger than the iteratively determined one. The standard deviations indicate that especially for the no tax class the multipliers vary strongly.

	Average Multiplier	StdDev
CARA (no tax)	5.547	3.232356
CARA (starting tax)	3.322	2.653128
CARA (basic tax)	2.423	2.196298
CARA (higher tax)	0.352	0.885589

Table 14 Granger Causality Test: Sales <> Risk Coefficients 1969M11-2006M04. The F-statistics show that there is no Granger Causality detectable.

		ΔX does not cause ΔSales		ΔSales does not cause ΔX	
Variable X	**Lag length**	**F-statistic**	**p-value**	**F-statistic**	**p-value**
CARA basic tax	1	0.03210	0.8579	0.05488	0.8149
	3	0.11387	0.9491	0.07460	0.9736
	6	0.08073	0.9980	0.07130	0.9986
CRRA basic tax	1	0.00122	0.9722	0.13703	0.7114
	3	0.26026	0.8540	0.14748	0.9313
	6	0.14386	0.9902	0.12674	0.9930
CRRA basic tax with wealth	1	0.00397	0.9498	0.10389	0.7474
	3	0.25575	0.8572	0.17186	0.9154
	6	0.14424	0.9901	0.15179	0.9887
CRRA higher tax with wealth	1	0.00022	0.9883	0.00348	0.9530
	3	0.08129	0.9702	0.68070	0.9769
	6	0.05351	0.9994	0.03847	0.9998

Table 15 Potential macroeconomic and bond factors influencing sales.

Factor	**Short-run relationship (Granger Causality)**	**Long-run relationship (Cointegration)**
FTSE100	unverifiable	unverifiable
PB interest rate	unverifiable	unverifiable
UK unemployment rate	unverifiable	unverifiable
PB interest rate/IB interest rate ratio	unverifiable	unverifiable
Variance PB prize distribution	unverifiable	unverifiable

Table 16 Granger Causality Test: Sales <> Skewness 1969M10-2006M04.

	Δ(Skewness) does not cause Δ(Purchases)		Δ(Purchases) does not cause Δ(Skewness)	
Lag length	**F-statistic**	**p-value**	**F-statistic**	**p-value**
1	0.11047	0.73977	1.88416	0.17057
2	0.54191	0.58203	2.04002	0.13128
3	0.71960	0.54069	1.68280	0.17000
4	1.79186	0.12946	1.59816	0.17383
5	2.15908	0.05772*	1.37831	0.23125
6	1.79798	0.09804*	3.24024	0.00401***

* significant at 10%

*** significant at 1%

Table 17 Johansen Cointegration Test: Sales <> Skewness 1970M03-2006M04.

	Hypothesized No. of CE(s)	Trace Statistic	Max-Eigen Statistic	p-value[1]
Trace Test	None	25.13511	-	0.0013***
	At most 1	0.259313	-	0.6106
Maximum Eigenvalue	None	-	24.8758	0.0008***
	At most 1	-	0.259313	0.6106

1) MacKinnon-Haug-Michealis (1999) p-values.
Trend Assumption: Linear deterministic trend
*** significant at 1%

Table 18 Johansen Cointegration Test: Sales <> Maximum Investment. 1980M01-2006M04.

	Hypothesized No. of CE(s)	Trace Statistic	Max-Eigen Statistic	p-value[1]
Trace Test	None	23.32294	-	0.0027***
	At most 1	0.089384	-	0.7650
Maximum Eigenvalue	None	-	23.23356	0.0015***
	At most 1	-	0.089384	0.7650

1) MacKinnon-Haug-Michealis (1999) p-values.
Trend Assumption: Linear deterministic trend
*** significant at 1%

Table 19 OLS Regression: Dependent Variable: Sales 1993M10-2006M04.

Independent Variable	Coefficient	Std.Error	t-statistic	p-value
Skewness	3277.993	378.9807	8.649497	0.0000
MPDummy	3.21E+08	34097818	9412010	0.0000
MaxInvest	81.11738	1723.019	0.047079	0.9625
Adjusted R-squared	0.69586		Durbin-Watson stat	1.13345
S.E. of regression	98954141		Observations	151

Table 20 ARIMA: Dependent Variable: Delta(Sales) 1993M10-2006M04.

Independent Variable	Coefficient	Std.Error	t-statistic	p-value
Delta(Skewness)	-2406.263	1211.000	-1.987005	0.0488
Delta(MPDummy)	2.44E+08	26793653	9.095951	0.0000
Delta(MaxInvest)	39217.77	7818.787	5.015838	0.0000
AR(1)	-0.683868	0.083093	-8.23013	0.0000
AR(2)	-0.217211	0.091591	-2.371529	0.0190

Adjusted R-squared	0.490814	Durbin-Watson stat	2.00205
S.E. of regression	71861620	Observations	151

Figures

Figure 1 Monthly Net Contribution. The figure shows the monthly net contribution as the difference between sales and repayments from October 1969 to April 2006. Before this time, sales figures are only available annually.

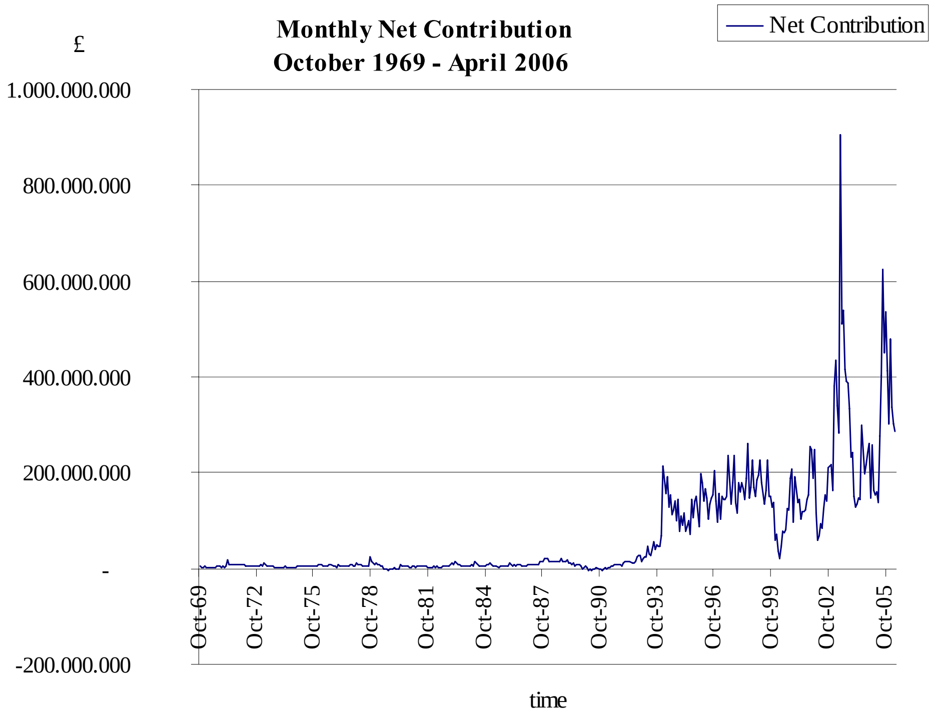

Figure 2 Indifference Risk Coefficients. Investors who exhibit individual risk coefficients smaller than α (shaded area) will decide to purchase Premium Bonds. In this case the CARA or CRRA utility is higher than the utility of an alternative risk-free investment.

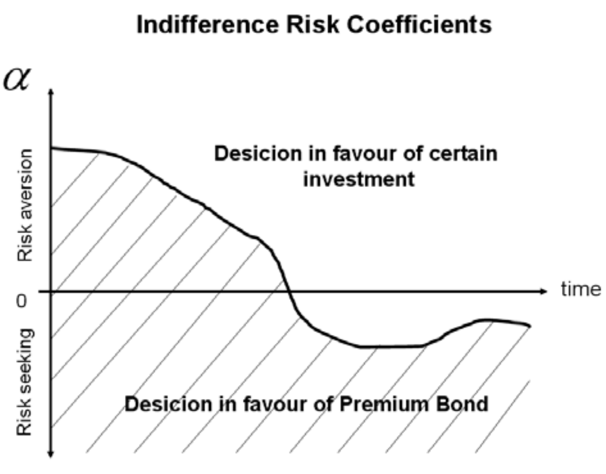

Figure 3 BoE Rate and Income Bond vs. Premium Bond. The figure shows the interest rates of the Premium Bond (PB), the Bank of England Rate (BoE) and the Income Bond (IB, invested capital assumed less than £25,000). From 1957 to 1995 the BoE rate was quite volatile. The Premium Bond was only rarely adjusted to the current interest level, and in some phases the margin of interest was very large (e.g. in 1980) or even a bit negative (e.g. in 1963 or 1977). Since 1995, NS&I seems to link the PB more tightly to the BoE rate and therefore the margin is kept relatively constant. In terms of the Income Bond, one can state that its interest rate has always been higher than the Premium Bond. With the exception of a huge margin around 1990, the margin stayed relatively constant on a lower level.

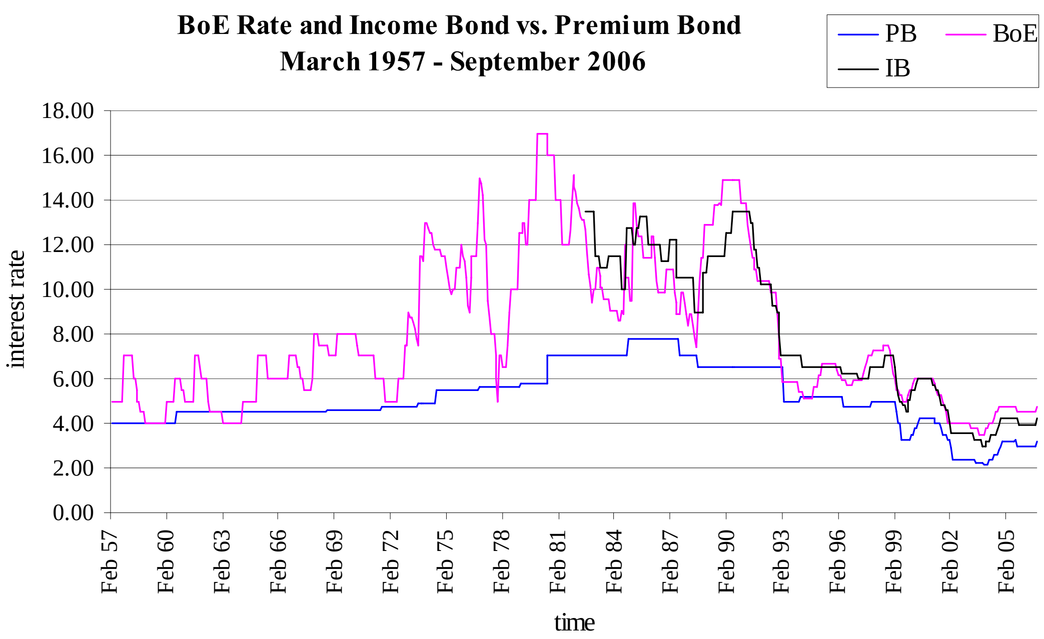

Figure 4 CARA: Premium Bond vs. BoE Rate for higher (H) and basic (B) rate taxpayers. In some years when the margin of interest between the Premium Bond and the BoE rate was virtually zero or even negative, the CARA risk coefficients became most positive. This was the case in November 1977. In comparison to the risk coefficients of the higher rate taxpayers (CARA(H)) the risk coefficients of the starting rate and non-taxpayers CARA(S) and CARA(0) are very close to zero and therefore not plotted.

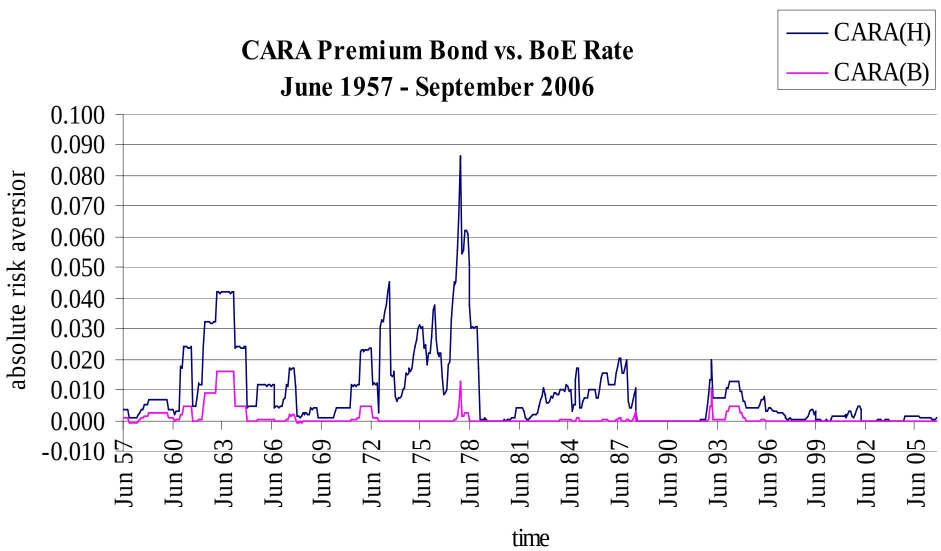

Figure 5 CRRA: Premium Bond vs. BoE Rate. Analysing the relative risk aversion points out the volatility of the risk measures. For higher rate taxpayers, even a certain degree of risk taking has been sufficient. The most remarkable exception can be identified in 1990 when the Bank of England base rate was set to 13.88% and the Premium Bond only yielded 6.5%. At this time, even higher rate taxpayers had to be risk seeking. In contrast, people who are not liable to tax or have not yet used up their personal tax allowance virtually always needed to be risk seekers.

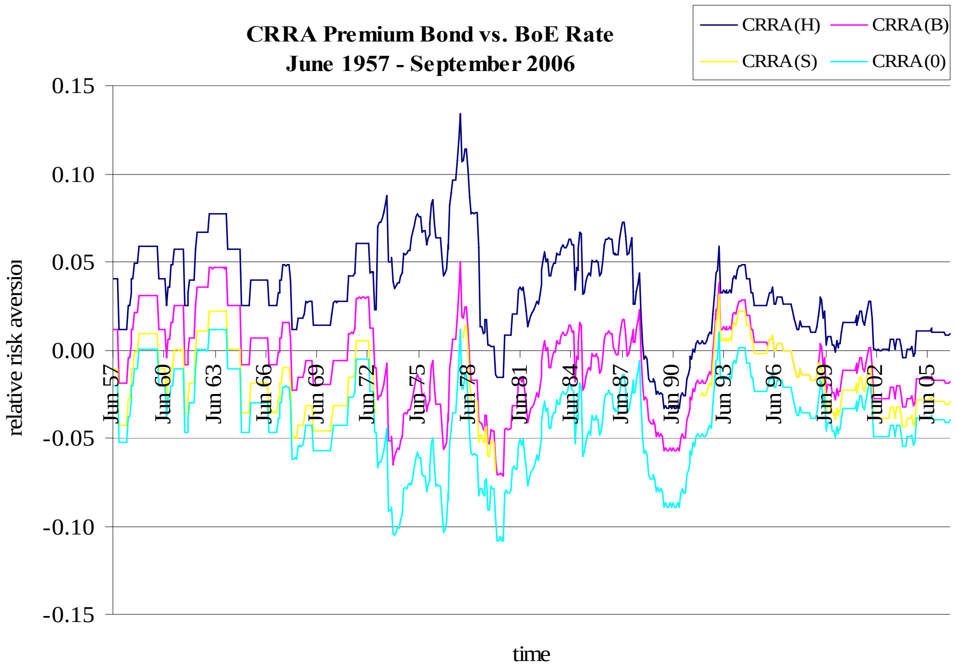

Figure 6 Monthly Spot Rates with maturity January 1999. The trend line points out that the spot rates decreased from 1979 to 1999.

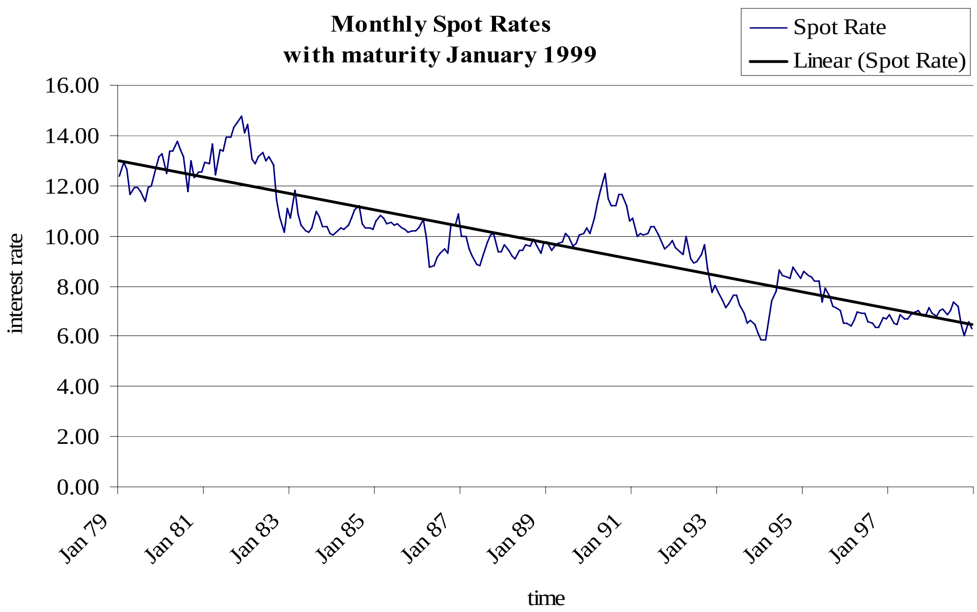

Figure 7 Premium Bond monthly sales. The arrows mark eye-catching peaks in sales. From 1969 till the end of 1991 the monthly sales moved on a steady level of about £14.8 million. Then they started to increase slowly and finally, by the beginning of 1994, shoot up. Additional peaks can be observed in May 2003 and August 2005.

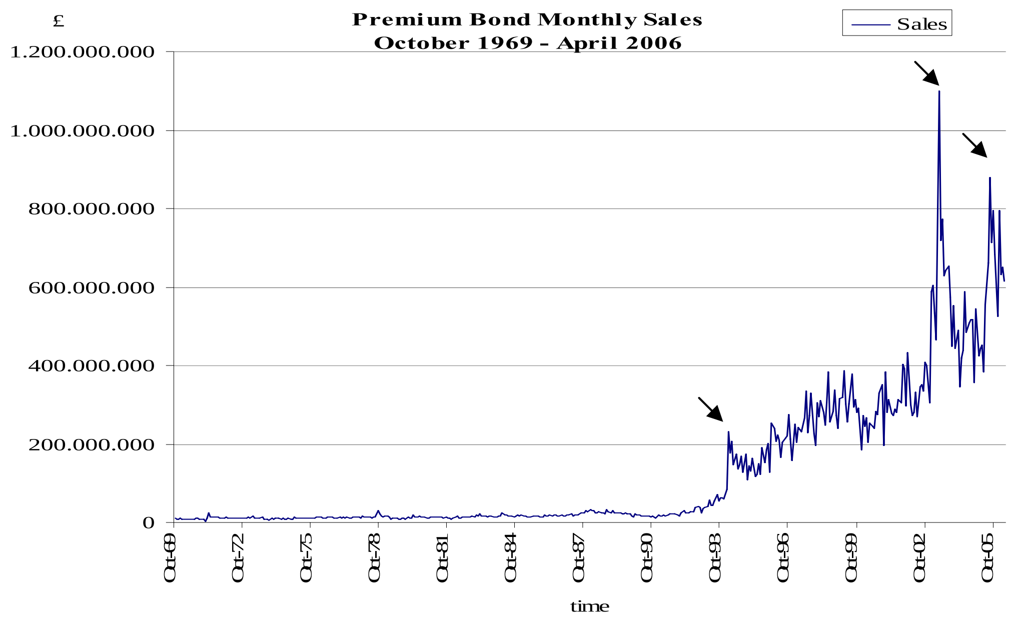

Figure 8 Premium Bond Total Stock. Despite the repayments, the net contribution is strongly positive and therefore the Premium Bond total stock has been growing especially since 1993.

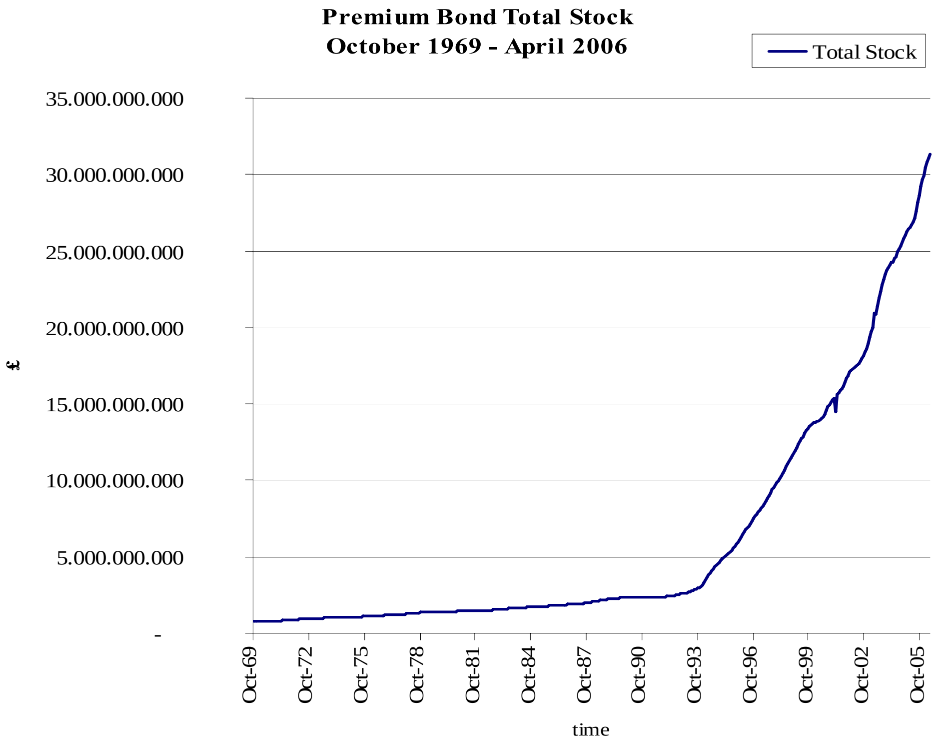

Figure 9 Skewness of prizes compared with sales. The graph shows that since 1957 the skewness of prize distribution has increased strongly. The development of skewness exhibits roughly the same trend pattern as the development of sales. In 1993, the introduction of the one million pounds jackpot clearly increased skewness and sales.

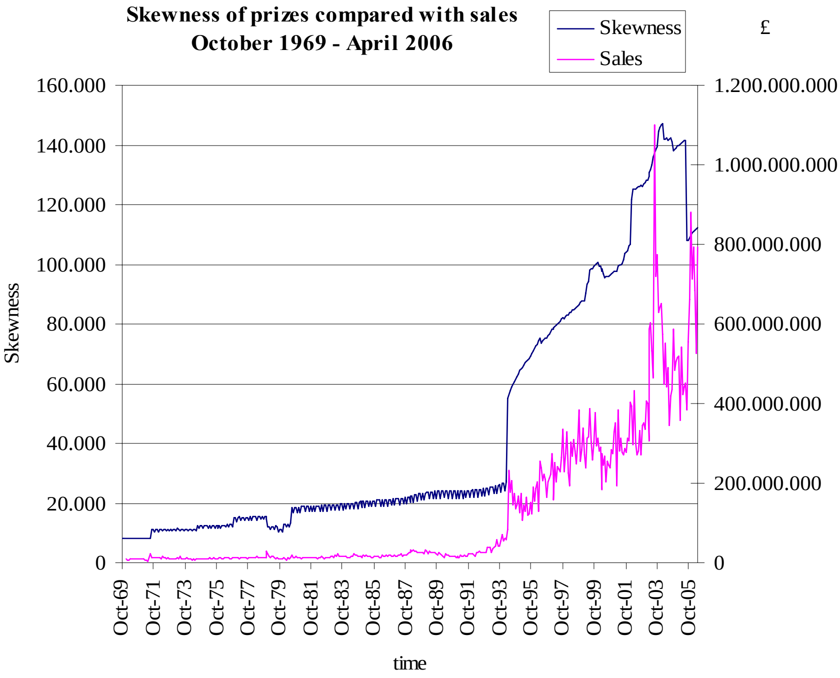

Figure 10 Static Forecast of Sales, December 2001 – April 2006. The Mean Absolute Percentage Error is 14.43 and the Theil Inequality Coefficient amounts 0.088657.

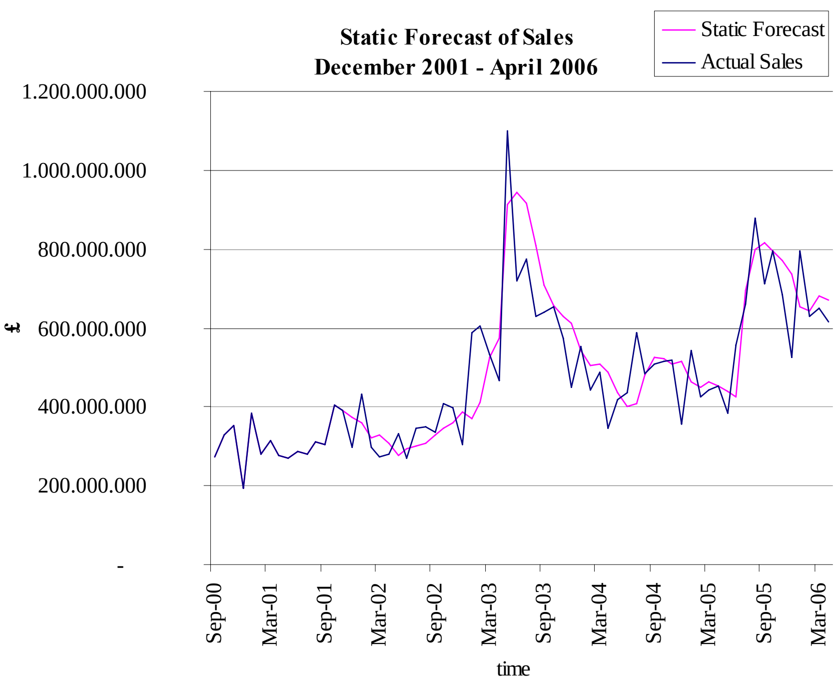

Figure 11 Dynamic Forecast of Sales, Dummy variable differenced, December 2001 – April 2006**.** The ARIMA model overestimates especially towards 2005. The Mean Absolute Percent Error in this case is 34.732 and the Theil Inequality Coefficient 0.175465.

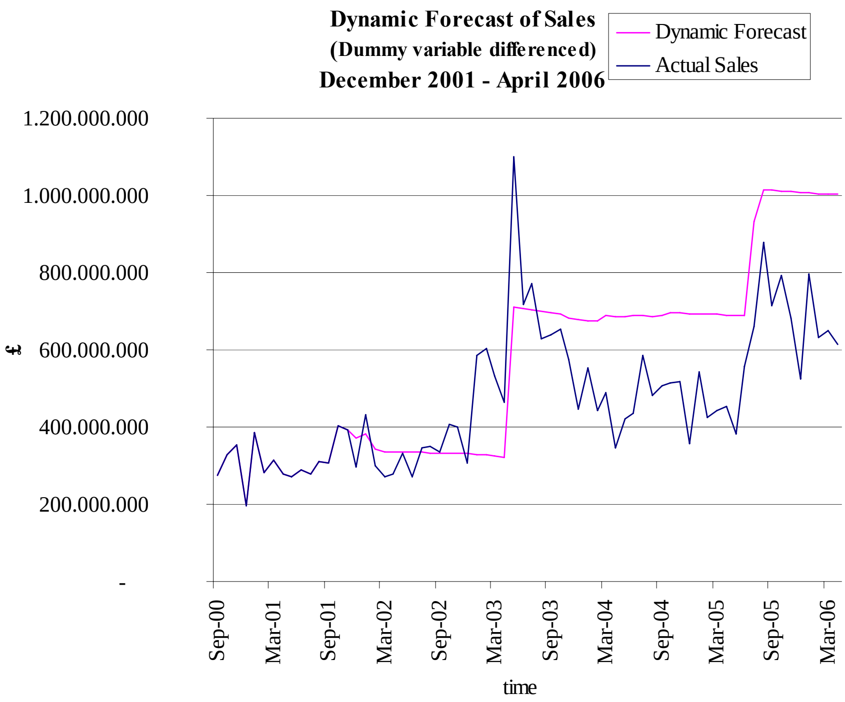

Figure 12 Dynamic Forecast of Sales, Dummy variable not differenced, December 2001 – April 2006. Not differencing the dummy variable and recalculating the model clearly produces a better forecast. The Mean Absolute Percent Error is now 25.729 and the Theil Inequality Coefficient 0.139428.