The Optimal Demand for Retail Derivatives

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Abstract

It has been shown that investors can benefit from including derivatives into their portfolios. For retail investors, however, a direct investment in derivatives is often too complicated. Investment certificates offer a potential solution to this problem and may help the investor to come closer to a complete market. We analyze if retail investors who buy and hold their portfolio for one year can indeed benefit from an investment in these certificates. We use a model with stochastic volatility and jumps calibrated to the German stock market index DAX. We find that the benefit of investing in typical retail products is equivalent to an annualized risk-free excess return of at most 35 basis points for a CRRA investor with a low risk aversion. If we take transaction costs into account, this number reduces to at most 14 bp. In terms of the types of contracts, we find that discount certificates perform best, while more sophisticated certificates, in particular those with knock-in or knock-out features, should often not be held by investors at all. Therefore, standard preferences cannot explain the large observed demand for investment certificates.

Keywords: Asset Allocation, Risk Premia, Structured Products, Discrete Trading, Calibration of Jump-Diffusion Models

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1 Introduction

Investment certificates and retail derivatives are exotic options or contracts which combine an investment into a stock or bond with plain vanilla or exotic options and which are offered to retail investors. They enlarge the set of payoff profiles available to these investors. Over the last years, these contracts have become more and more popular. In 2006, investment certificates with a total value of 69.17 billion € were turned over in Germany (Deutsches Derivate Institut (2007)), compared to a total investment by Germans in the stock market of 372 billion € (Bundesverband Deutscher Banken (2007)). 30.5% of the traded certificates were bonus certificates, 29% index and participation certificates and 29% discount certificates.

In this paper, we analyze whether and to which extent retail investors benefit from trading these investment certificates. We consider rational investors who maximize their expected utility from terminal wealth, and we focus on an economy with both stochastic volatility and jumps in stock prices.

Investors can increase their utility significantly by trading plain vanilla options, which is documented both theoretically (e.g. Liu and Pan (2003) or Branger, Schlag, and Schneider (2008)) and empirically (e.g. Jones (2006) or Driessen and Maenhout (2007)). In contrast to these papers, our focus is on investment certificates. The benefits from trading reverse discount products are analyzed by Breuer and Perst (2006) who consider investors who behave according to prospect theory in a Black-Scholes setup. Different from their analysis, we assume that investors maximize their expected utility in a market with stochastic volatility and jumps.

In a complete market and with continuous trading, an investor can achieve the overall optimal future payoff profile by investing into the stock, the risk-free bond, and sufficiently many (maybe even infinitely many) derivatives. In this setup, the exact payoff profiles of the derivatives do not matter, and the investor is indifferent between trading plain-vanilla options or more exotic contracts. There would thus be nothing special about retail derivatives. In reality, however, trading takes place in discrete time, and markets are in general incomplete. Furthermore, retail investors are hindered from implementing the optimal portfolio strategies due to too high minimum investment amounts, high transaction costs or margin requirements, short-selling restrictions, and maybe also a lack of knowledge. In this situation, the characteristics of the available derivatives may actually matter a lot.

A retail investor will then benefit from a derivative whose payoff profile is equal (or close) to his[1] optimal payoff profile and which is offered by an issuer who is better than himself able to implement the corresponding replication strategy. The optimal payoff profile, however, is in general highly complicated and does not only depend on observable stock prices, but also on the paths of state variables like stochastic volatility. Furthermore, it is specific to the investor under consideration. Due to these reasons, these optimal payoff profiles will not be traded in the market. Nevertheless, financial institutions might design contracts that at least approximate these optimal payoffs and help the investors to come closer to their optimal payoffs. If investment certificates are such contracts, they should increase the utility of the investor significantly, and there should thus be a significant demand for these contracts. Stated differently, ideal retail derivatives would help the investor to come close to a complete market, and the question is how well the most popular investment certificates achieve this objective of market completion.

[1]Females are always included.

Our analysis focuses on the most heavily traded types of contracts, which are also those that have been present in the market for the longest time. Discount certificates combine an investment in the stock with a short position in a call. In exchange for capping the participation in stock price gains to a certain level, these certificates trade at a discount compared to the underlying. They are offered with different threshold levels to match the preferences of different types of investors. A new variant of these contracts are rolling discount certificates, which automatically roll over discount certificates with a given time-to-maturity and a given moneyness into new discount certificates with the same characteristics. The risk profile of these contracts is much more stable over time than for standard discount certificates. Sprint certificates are another variant of retail derivatives. If the stock price ends up in a certain corridor, the investors participate with a factor of two in any gains and losses, while the participation rate is one for levels below this corridor and zero above. Another popular contract are bonus certificates, which promise a fixed payoff yielding a return above the risk-free rate if the underlying does not fall below a lower threshold during the lifetime of the contract. Otherwise, the investor receives the stock. Finally, turbos (or mini-futures) exist as so-called 'long' and 'short' versions, which are equivalent to down-and-out calls and up-and-out puts, respectively. They coincide with normal call (put) options as long as a lower (upper) threshold is not crossed during the lifetime of the contract, and become worthless otherwise.

We analyze whether an investor benefits from having access to these retail derivatives by comparing his optimal utility with and without these instruments. We consider an investor with standard CRRA utility who follows a buy-and-hold strategy with a horizon of one year. Since we deal with retail investors, we include short-selling restrictions and do not allow borrowing at the risk-free rate. The stock follows a jump-diffusion process with stochastic volatility as in Bates (2000), with the only exception that we use one stochastic volatility component instead of two. Broadie, Chernov, and Johannes (2008) show that such an option pricing model with non-zero risk premia is able to generate empirically observed expected (put) option returns. It should thus describe the investment opportunity set in a rather realistic way. We calibrate this model to data for the German stock market index DAX and the associated index options. With discrete trading and a finite number of derivatives only, the optimal investment strategy in this model cannot be found in closed form. We rely on a numerical optimization as in Branger, Breuer, and Schlag (2007) to determine the optimal portfolios.

We find that a CRRA investor who holds a portfolio of the stock, discount certificates and the money market account without rebalancing for one year can earn an annualized excess certainty equivalent return of up to 35 basis points compared to a buy-and-hold strategy without certificates. Sprint certificates offer a utility improvement which is only slightly smaller. The gain from rolling discount certificates depends on the maturity of the underlying certificates, with at most 21 bp for a maturity of two years and at most 11 bp if certificates with one month to maturity are rolled over. Path-dependent contracts with knock-in and knock-out features, like bonus certificates and turbos, perform very poorly, and the same holds true for guarantee certificates. The utility gain is at most 8 bp, and for long and short turbos and guarantee certificates, the CRRA investor does not even invest into them at all (or would rather like to take a short position).

As a robustness check, we also use loss aversion utility of Tversky and Kahnemann (1992). The overall results remain the same, and again discount certificates (even with the same strike price as for CRRA utility) yield the largest utility improvement. The only major difference to CRRA utility is that an investor with loss aversion now profits from a long position in guarantee certificates, even if the utility gain of 4.3 bp is rather small.

In the analysis described so far, we assume that the investor can buy retail derivatives at the model price. In reality, however, retail derivatives come at a cost. They are not freely traded but can only be bought from and sold back to the issuer. Since price differences cannot be arbitraged away, the prices of these contracts are often not equal to the price of the replicating portfolio before expiration. For the German market, Wilkens, Erner, and Röder (2003), Stoimenov and Wilkens (2005), Muck (2006), Wilkens and Stoimenov (2007), and Muck (2007) provide some empirical evidence on this hypothesis. We summarize these price deviations more generally as transaction costs.

In the second step of our analysis, we take these transaction costs into account. We restrict this analysis to discount certificates and sprint certificates, which performed best in the first step. Again, we numerically determine the optimal portfolio and compute the utility gain of an investor who has access to derivatives. The results change significantly. The only contracts the investor still wants to hold are discount certificates, with an additional certainty equivalent return of at most 14 bp. Sprint certificates are not attractive for the investor anymore, since the transaction costs are larger than the potential benefits the contract might have for the investor. Thus, the investor prefers to invest his wealth into the stock and money market account only.

The reminder of the paper is organized as follows. The option pricing model and the optimization problem of the investor are introduced in Section 2, while Section 3 describes the methodology used to calibrate the model. Section 4 shortly discusses the investment certificates the investor has access to. The results of the numerical analysis are discussed in Section 5. Section 6 concludes.

2 Model Setup

2.1 Option Pricing Model

We consider an option pricing model with stochastic volatility and stochastic jumps in returns. Following Bates (2000), we assume that the jump intensity is proportional to the diffusion variance of the stock. The relative contribution of the diffusion component and the jump component to the overall variance of the stock price is thus constant over time. The dynamics of the stock price and volatility under the physical measure $\mathbb{P}$ are given by

$$dS_t = \left(\mu_t - E^{\mathbb{P}}\left[e^{X_t} - 1\right]\lambda^{\mathbb{P}} V_t\right) S_t dt + \sqrt{V_t} S_t dB_t^1 + \left(e^{X_t} - 1\right) S_{t-} dN_t \quad (1)$$

$$dV_t = \kappa^{\mathbb{P}}\left(\bar{v}^{\mathbb{P}} - V_t\right) dt + \sigma_V \sqrt{V_t}\left(\rho dB_t^1 + \sqrt{1-\rho^2} dB_t^2\right), \quad (2)$$

where the second term in the first equation is the compensator of the jump term. B_t^1 and B_t^2 are independent Wiener processes. The jump intensity of the Poisson process N is $\lambda^{\mathbb{P}} V_t$, and jumps in log returns are assumed to be normally distributed:

$$X_t \sim N\left(ln\left(1 + \mu_X^{\mathbb{P}}\right) - 0.5\left(\sigma_X^{\mathbb{P}}\right)^2, \left(\sigma_X^{\mathbb{P}}\right)^2\right).$$

The expected excess return of the stock is

$$\mu_t = r + \eta_1 V_t + E^{\mathbb{P}}\left[e^{X_t} - 1\right]\lambda^{\mathbb{P}} V_t - E^{\mathbb{Q}}\left[e^{X_t} - 1\right]\lambda^{\mathbb{Q}} V_t,$$

where $\eta_1 V_t$ is the risk premium for diffusive stock price risk $\sqrt{V_t} dB_t^1$, and $\lambda^{\mathbb{Q}} V_t$ is the intensity of the jump process under the risk-neutral measure.

The dynamics under the risk-neutral measure $\mathbb{Q}$ are

$$\begin{aligned} dS_t &= \left(r - E^{\mathbb{Q}}\left[e^{X_t} - 1\right]\lambda^{\mathbb{Q}} V_t\right) S_t dt + \sqrt{V_t} S_t d\widetilde{B}_t^1 + \left(e^{X_t} - 1\right) S_{t-} dN_t \\ dV_t &= \kappa^{\mathbb{Q}}\left(\bar{v}^{\mathbb{Q}} - V_t\right) dt + \sigma_V \sqrt{V_t}\left(\rho d\widetilde{B}_t^1 + \sqrt{1-\rho^2} d\widetilde{B}_t^2\right), \end{aligned}$$

where $\widetilde{B}^1$ and $\widetilde{B}^2$ are again independent Wiener processes. The jump sizes in log returns under the risk neutral measure are normally distributed:

$$X_t \sim N\left(ln\left(1 + \mu_X^{\mathbb{Q}}\right) - 0.5\left(\sigma_X^{\mathbb{Q}}\right)^2, \left(\sigma_X^{\mathbb{Q}}\right)^2\right).$$

In the following, we assume that the jump risk premium depends only on the differences in the mean jump size $\mu_X^{\mathbb{P}}$ and $\mu_X^{\mathbb{Q}}$, while the volatility of the jump size and the jump intensity are equal under both measures, so that $\sigma_X^{\mathbb{P}} = \sigma_X^{\mathbb{Q}} \equiv \sigma_X$ and $\lambda^{\mathbb{P}} = \lambda^{\mathbb{Q}} \equiv \lambda$.

The relation between the volatility processes under the physical and risk neutral measure is given by

$$\kappa^{\mathbb{P}} \bar{v}^{\mathbb{P}} = \kappa^{\mathbb{Q}} \bar{v}^{\mathbb{Q}} \tag{3}$$

and

$$\kappa^{\mathbb{Q}} = \kappa^{\mathbb{P}} + \sigma_V\left(\rho\,\eta_1 + \sqrt{1-\rho^2}\,\eta_2\right), \tag{4}$$

where $\eta_2 V_t$ is the risk premium for pure volatility diffusion risk $\sqrt{V_t} dB_t^2$. $\eta_V = \kappa^{\mathbb{Q}} - \kappa^{\mathbb{P}}$ is the overall volatility risk premium.

2.2 Investor

We consider an investor with constant relative risk aversion γ. He derives utility from terminal wealth at time T only, and he maximizes expected utility

$$E\left[\frac{W_T^{1-\gamma}}{1-\gamma}\right].$$

This investor can invest into the underlying stock and the money market account with a constant risk-free interest rate r. Furthermore, he has access to some derivatives which we will specify in more detail in Section 4.

Since we consider the portfolio problem of a retail investor, we impose two restrictions on his portfolio policy. First, we assume that he cannot trade continuously, but rather follows a buy-and-hold strategy. His terminal wealth at time T is given by

$$W_T = W_0 \left[e^{rT} + \phi_0 \left(\frac{S_T}{S_0} - e^{rT} \right) + \sum_{i=1}^{n} \psi_0^i \left(\frac{C_T^i}{C_0^i} - e^{rT} \right) \right],$$

where ϕ_0 denotes the relative portfolio weight of the stock at time $t = 0$. ψ_0^i is the relative portfolio weight of the i-th derivative ($i = 1, \ldots, n$) with price C_t^i at time t. Second, we impose short selling constraints on the index, the derivatives, and the money market account when we analyze an investment in certificates. The latter restriction is motivated by the fact that a retail investor will not be able to borrow at the risk-free rate.

In the analysis of the portfolio decisions, we will also consider the exposure of the optimal portfolio to the different risk factors (see also Liu and Pan (2003)). Starting from the portfolio weights and the sensitivities of the assets, the dynamics of the investor's wealth can be rewritten as

$$\begin{aligned} dW_t &= rW_t dt + \theta_t^{B1} W_t \left(\eta_1 V_t dt + \sqrt{V_t} dB_t^1 \right) + \theta_t^{B2} W_t \left(\eta_2 V_t dt + \sqrt{V_t} dB_t^2 \right) \\ &\quad + W_{t-} \left(\theta_{x\%,t}^N dN_t - E_t^{\mathbb{Q}} \left[\theta_{x\%,t}^N \right] \lambda^{\mathbb{Q}} V_t dt \right), \end{aligned}$$

where θ_t^{B1} and θ_t^{B2} are the relative exposures to stock diffusion risk $\sqrt{V_t} dB_t^1$ and pure volatility diffusion risk $\sqrt{V_t} dB_t^2$, respectively. $\theta_{x\%,t}^N$ captures the percentage change in the investor's wealth if a jump equal to the $x\%$-quantile of the jump size distribution occurs.

The portfolio planning problem cannot be solved in closed form, but we have to rely on numerical optimization. Details can be found in Branger, Breuer, and Schlag (2007) and are summarized in Appendix A. Different from the optimization there, we do not set the initial variance equal to the long-run mean, but rather perform the optimization for a whole grid of initial variance levels. The unconditionally expected utility is then a weighted average of the expected utilities for the different variance levels, where the weights are given by the distribution of the variance process running for one year and started with a variance equal to the long run mean.[2]

2.3 Measuring Utility Gains

An investor who has access to derivatives will be at least as well off as an investor who cannot trade in these contracts. To measure his utility gain, we compare the certainty equivalents in both cases. If the investor has access to derivatives, his indirect utility is denoted by J_w, and the certainty equivalent wealth $\mathcal{W}_w$ is defined by

$$J_w = \frac{1}{1-\gamma} \mathcal{W}_w^{1-\gamma}.$$

In the case without derivatives, the indirect utility and the certainty equivalent wealth are $J_{w/o}$ and $\mathcal{W}_{w/o}$, respectively. The utility gain is then captured by the annualized log difference between $\mathcal{W}_w$ and $\mathcal{W}_{w/o}$:

$$R_{w/o \to w} = \frac{\ln(\mathcal{W}_w / \mathcal{W}_{w/o})}{T},$$

where T is the investment horizon. $R_{w/o \to w}$ can be interpreted as an additional certainty equivalent return.

[2]The distribution of the variance after one year is already very close to the stationary distribution of the variance. We also performed the analysis using the distribution after 10 years, and the results were nearly identical.

3 Calibration

To work with a realistic structure of option prices and with realistic risk premia for the German market, we calibrate the model to the German stock market index DAX. Note that we do not aim at a full-grown estimation of the model here. For the portfolio planning problem, we need the parameters both under the $\mathbb{P}$- and under the $\mathbb{Q}$-measure, for which we basically rely on time-series information and on the cross-section of option prices, respectively.

Our sample consists of daily settlement prices for European put options on the DAX with a wide range of strikes and maturities for the time period from January 1997 to December 2004.[3] To reduce the number of option price observations to a manageable amount, we only use the prices at the 10th day of each month or the next banking day. Observations with prices of 0.10 € (the minimal price possible), with 5 calendar days or less until expiration, with strikes below 90% and above 110% of the current index level (to insure an approximately equal weight of in-the-money and out-of-the-money options) and with more than 2 years until expiration are excluded. This leaves us with 9,345 observations. Daily observations on the XetraDax, the version of the DAX calculated based on trades on the electronic trading platform Xetra, and daily interest rates for different maturities are obtained from Datastream. Interest rates are interpolated linearly to match the maturities of the respective options.

We start the calibration by obtaining (some of the) parameters under the $\mathbb{P}$-measure. The equity risk premium of 5.5% for the DAX is calculated from a time series of the DAX over a longer time period from January 1975 to December 2006, since eight years

[3]We thank Deutsche Börse for providing the data and Patrick Behr for help in obtaining it.

of data only would not give a reliable estimate. We then identify the large jumps that have occurred over the eight year period from 1997 to 2004.[4] Given the diffusion variance (which we initially set equal to the annualized volatility of 20% of monthly DAX returns from 1997 to 2004), we determine a 99%-confidence interval for daily stock price changes generated by the diffusion component only, and classify every daily stock price move that leaves this interval as a (large) jump. These jumps are taken out of the time series of daily return observations and are replaced by a one standard deviation movement in the same direction. The resulting time series is then 'generated' by the diffusion only, and we estimate the local diffusion variance for each day from the returns of the 20 preceding trading days. With these estimates for the diffusion variance and now monthly data, we repeat the identification of jumps. This leaves us with seven (large) jumps in our eight year observation period, 3 upward and 4 downward jumps. The largest upward jump in monthly excess returns is +19.9% in April 2003, the largest downward jump is -20.9% in September 2002. To determine the mean and the variance of the jump size, we then rely on a simulation study. For a given set of candidate parameters, we simulate a time series of index returns, apply the identification methodology just described, and determine the mean and volatility of the identified jumps. These simulated values have to be equal to the mean and volatility found in the data, which results in a mean jump size μ_X^P of -1.8% and a volatility of the jump size σ_X of 11%.

To estimate the jump frequency, we calculate for every month, based on the jump size distribution and the local diffusion volatility, the probability that a jump is too small

[4] Our approach is similar to Tauchen and Zhou (2006), who rely on the difference between bi-power variation and realized variance to identify realized jumps in high-frequency intraday data, while most other papers treat jumps as latent variables.

to be detected. This probability is then averaged over all months in the sample period. We find that 75% of all jumps remain undetected. With 7 observed jumps in 8 years, this yields an average of $\frac{0.875}{1-0.75} = 3.46$ jumps per year.

The variance of monthly returns for the time period 1997-2004 is 0.0058. We set this variance equal to the expected variance of local stock price changes, which is in turn decomposed into the diffusion variance and the variance coming from the jump component:

$$\begin{aligned} E^{\mathbb{P}}\left[Var\left(\frac{dS_t}{S_{t-}} \,\Big|\, V_t\right)\right] &= E^{\mathbb{P}}\left[V_t dt + \left((1+\mu_X^{\mathbb{P}})^2 e^{\sigma_X^2} - 2\mu_X^{\mathbb{P}} - 1\right)\lambda V_t dt\right] \\ &= \bar{v}^{\mathbb{P}} dt + \left((1+\mu_X^{\mathbb{P}})^2 e^{\sigma_X^2} - 2\mu_X^{\mathbb{P}} - 1\right)\lambda \bar{v}^{\mathbb{P}} dt. \end{aligned} \tag{5}$$

With monthly returns and therefore $dt = \frac{1}{12}$, this yields $\bar{v}^{\mathbb{P}} = 0.028$. Since the average jump intensity is given by $\lambda\bar{v}^{\mathbb{P}}$, the resulting jump frequency parameter λ can be calculated as $\frac{3.46}{\bar{v}^{\mathbb{P}}}$.

The next step of the calibration builds on the cross-section of option prices. The parameters $\kappa^{\mathbb{Q}}$, $\bar{v}^{\mathbb{Q}}$, $\mu_X^{\mathbb{Q}}$, σ_V, ρ and a local variance for each of the 96 days in the sample are chosen such that the sum of squared differences in Black-Scholes implied volatilities between the model prices and the market prices is minimized. We impose the additional restrictions that $\bar{v}^{\mathbb{Q}}$ is equal to the average of the calibrated local variances, and that the volatility of volatility σ_V and the correlation ρ are equal to the values estimated from the time series of calibrated volatilities. Given these parameters, $\kappa^{\mathbb{P}}$ is then obtained from Equation (3).

The final step is to find the market prices for stock diffusion risk and volatility diffusion risk. The equity risk premium of 5.5% can be decomposed into the average compensation $\eta_1\bar{v}^{\mathbb{P}}$ for diffusion risk and the average jump risk premium of $\lambda\bar{v}^{\mathbb{P}}(\mu_X^{\mathbb{P}} - \mu_X^{\mathbb{Q}}) = 3.1\%$. From this relation, we obtain η_1. The parameter η_2 then follows from Equation (4).

The final parameter values can be found in Table 1. Comparing our values with the results of Guse and Muck (2006), who do an implied state-GMM estimation of an option pricing model on the DAX using a similar sample period, but do not estimate volatility risk premia separately, we find that our calibration yields more jumps (approximately 3.5 per year versus 0.5 in their study) and a lower average diffusion variance. Our jump risk premium of 3.1%, however, is similar to their estimate, which ranges from 2.3% to 4.3% depending on the data used. Eraker (2004) even finds a jump risk premium of 7.8%. The volatility risk premium of $\eta_V = -0.4$ is rather small compared to Eraker (2004), who finds a volatility risk premium of $\eta_V = -2.3$ and Pan (2002) who finds $\eta_V = -3.1$ for the S&P 500. Coval and Shumway (2001) and Bakshi and Kapadia (2003) find negative volatility risk premia as well. These two studies, however, do not quantify the volatility risk premium in a parametric way, but deduce its sign and size from the returns of portfolios of zero-beta straddles and delta-hedged option positions, respectively.

To assess the quality of our calibration, we provide some plots. Figure 1 shows the value of the VDAX volatility index, the calibrated diffusive volatilities, and the total calibrated volatility from jump and diffusion components. While the pure diffusive volatility lies below the VDAX, total volatility matches the VDAX very well. Figure 2 shows the differences in Black-Scholes implied volatilities between the model and the market as a function of moneyness and time to maturity, respectively. The calibrated model tends to underprice out-of-the-money puts and overprice in-the-money puts. This is due to the fact that we do not calibrate the correlation ρ to option prices, but estimate it from the time series of calibrated variances. Furthermore, the dispersion of the pricing errors is much larger for higher moneyness levels, which may reflect the fact that, at the right end of the moneyness spectrum, we use ITM puts instead of OTM calls. With respect to time-to-maturity, there is no specific pattern in the pricing errors besides the well-known phenomenon of a worse fit for very short times to maturity.

4 Contract Specifications

In this section, we briefly describe the investment certificates used.[5] Option prices are obtained via Fourier inversion using the method described in Duffie, Pan, and Singleton (2000), or, in case of path-dependent derivatives, via a Monte-Carlo simulation.

If the investor can trade only in the stock and the money market account, the exposure of his portfolio to pure volatility risk is zero, and the relation between the exposures to stock diffusion risk and jump risk is fixed. Investment certificates allow the investor to deviate from this fixed package offered by the stock (see also Liu and Pan (2003)) and thus help him to come closer to a complete market. He will then choose the certificate for which the risk profile best matches his overall optimal exposure (which he could only achieve with infinitely many derivatives and continuous trading). Since the risk exposure of the certificates is thus central to the asset allocation problem, we will also provide a short discussion of the sensitivities. Table 2 shows the delta and vega, as well as the price change $\Delta_{x\%}$ of the certificate for a jump in stock returns of a size equal to the x% quantile of the jump size distribution ($x = 5, 20, 80, 95$).

[5]Some of these certificates differ slightly from the corresponding certificates for stocks. For the latter, future dividends can be used to finance some special features of the payoffs, which is not possible if the underlying is a performance index like the DAX.

4.1 Discount Certificates

Discount certificates combine an investment in the index with a short position in a call option on the index. The payoff at maturity in T is

$$DC_T = S_T - \max(S_T - K, 0) = \min(S_T, K),$$

and the price at t is

$$p_t(DC_T) = S_t - C_t(S_t, K, T),$$

where $C_t(S, K, T)$ is the price at time t of a call with strike price K and maturity date T, when the current stock price is S. Since the payoff profile is concave, the vega of the discount certificate is negative. With a long position in the contract, the investor will thus earn a positive premium for volatility risk (given that the volatility risk premium is negative), as well as for jump risk (given that there is a positive premium for stock price jumps). The jump risk exposure is lower than for the stock. The lower the strike prices of the embedded options, the lower the exposures in absolute terms. The reduction in jump risk is largest, so that the ratio of volatility risk exposure to jump risk exposure is largest for low strike prices.

4.2 Sprint Certificates

Sprint certificates combine an investment in the index, a long position in a call with strike price K_1, and a short position in two calls with strike price $K_2 > K_1$. In practice, K_1 is often equal to the current value of the index, while K_2 is chosen such that the net costs for the three options are zero. In terms of the payoff, the investor participates by 200% in any stock price gains if the stock ends up in the interval $[K_1, K_2]$, he participates by

100% in the losses if the stock ends up below, and his payoff is capped at $K_1 + 2(K_2 - K_1)$ if the stock ends up above K_2. The price of the sprint certificate at time t is given by

$$p_t(SC_T) = S_t + C_t(S_t, K_1, T) - 2C_t(S_t, K_2, T).$$

For most empirically relevant parameter sets, the vega of the sprint certificate is negative. It will thus earn a positive premium for volatility risk, and it will also earn a positive premium for jump risk.

4.3 Guarantee Certificates

Guarantee certificates consist of an investment in a risk-free asset and a long position in call options on the index with a strike equal to the initial index level. The amount invested in the risk-free asset is chosen such that it result in the guaranteed payoff equal to 1.0 at maturity, and the remaining money is invested into the call options. The price at time t is

$$p_t(GC_T) = e^{-r(T-t)} \cdot 1.0 + \frac{1.0 - e^{-rT} \cdot 1.0}{C_0(S_0, S_0, T)} C_t(S_t, S_0, T).$$

Since the payoff profile of this contract is convex, the vega is positive. Therefore it will earn a negative premium for volatility risk. The premium for jump risk is positive.

4.4 Bonus Certificates

Bonus certificates on a performance index like the German index DAX are characterized by a bonus level BL above and a barrier BA below the current stock price. If the barrier is not crossed during the lifetime of the product, the investor receives BL. Otherwise, he gets the stock. The payoff is equal to

$$BC_T = \begin{cases} BL & \text{if } S_t > BA \text{ for all } 0 < t \leq T \\ S_T & \text{if } S_t \leq BA \text{ for some } 0 < t \leq T. \end{cases}$$

Since the payoff profile of this contract is in principle concave, vega will in most cases be negative, and the contract offers the possibility to benefit from negative volatility risk premia. Furthermore, bonus certificates earn a positive premium for jump risk.

4.5 Rolling Discount Certificates

Rolling discount certificates consist of discount certificates with short times to maturity, usually one month, which are rolled over at expiration into new discount certificates with the original moneyness and maturity. The rationale behind this contract design is to ensure a rather constant risk-return profile over time without the need for the investor to do the roll-over – at high transaction costs – on his own. The initial price of the rolling discount certificate is the price of a standard discount certificate with the desired maturity τ and moneyness k, $p_0(DC_\tau(kS_0)|S_0, V_0)$. The payoff of this certificate at time τ is then used to buy $s_1 = \frac{DC_\tau(kS_0)}{p_\tau(DC_{2\tau}(kS_\tau)|S_\tau, V_\tau)}$ units of the new certificate, and so on.

The risk exposure of rolling discount certificates is similar to that of standard discount certificates. Due to the much shorter time to maturity, however, vega is in most cases smaller in absolute terms (with the exception of ATM-options), while the jump risk exposure is larger. The ratio of volatility risk exposure to jump risk exposure is thus smallest for these contracts among all considered contracts except short turbos.

We also include rolling discount certificates for which the underlying discount certificates have a longer time to maturity, but which are nevertheless rolled over every month, i.e. long before they mature. To the best of our knowledge, these contracts are not yet traded, but turn out to be superior in our analysis later on.

4.6 Turbo Certificates

Long and short turbo certificates are technically equal to down-and-out calls and up-and-out puts, respectively. If a lower (upper) barrier BL (BU) is crossed, the turbo is knocked-out, and its payoff is zero. Otherwise, the payoff is equal to that of a standard call (put):

$$LT_T = \begin{cases} \max(S_T - K, 0) & \text{if } S_t > BL \text{ for all } 0 < t \leq T \\ 0 & \text{if } S_t \leq BL \text{ for some } 0 < t \leq T \end{cases}$$

$$ST_T = \begin{cases} \max(K - S_T, 0) & \text{if } S_t < BU \text{ for all } 0 < t \leq T \\ 0 & \text{if } S_t \geq BU \text{ for some } 0 < t \leq T \end{cases}$$

Long turbos have a positive delta, a negative vega and a negative exposure to downward stock price jumps. The vega is quite large in absolute terms for low volatilities and becomes small for large volatilities, while the jump risk exposure is rather constant across different volatility levels. The ratio of volatility to jump risk exposure is thus very unstable.

Short turbos have a negative delta, a positive vega, and a positive exposure to negative jumps. They thus react in the opposite way as most of the other certificates to the risk factors. Thus short turbos do not earn (but rather loose) volatility and jump risk premia.

5 Numerical Results

5.1 Optimal Portfolios and Utility Gains

We consider a buy-and-hold investor who can invest in the stock, the money market account, and one of the certificates discussed in Section 4. His investment horizon is one year. All certificates have a time-to-maturity of two years, with the exception of rolling discount certificates which are perpetual by construction, and bonus certificates and turbos, for which we also include contracts with a maturity of one year.

As benchmarks, we consider the case where the investor can invest only into the stock and the money market account, the case where he also has access to two additional plain vanilla options, and the ideal case of a complete market with infinitely many derivatives and continuous trading. For the benchmark cases, we drop the restrictions on short selling of derivative contracts. If some short positions turned out to be optimal, they could always be achieved by combining them with some long positions into a certificate in which the investor will then take a long position. This is for example the case for discount certificates, which include a short position in the call. Details on the numerical optimization routine for solving the portfolio planning problems in discrete time can be found in Branger, Breuer, and Schlag (2007) and are summarized in Appendix A. The optimal solution in a complete market is derived in Appendix B.

Table 3 shows the optimal portfolios and the utility gains due to derivatives, where the benchmark is a buy-and-hold strategy in the stock and the money market account. Since all results are qualitatively the same for all risk aversions, we restrict the discussion to the case $\gamma = 2$. The optimal positions in the assets and the initial exposures to the risk factors depend on initial volatility. The first number in Columns 2 to 6 represents the optimal portfolio weight or initial exposure for the lowest volatility level considered (5.5%), the second number is the optimal value for the highest volatility level (43.6%). For all certificates except rolling discount certificates with a maturity of one month of the underlying certificates, the investor puts a large fraction of his wealth into stocks and a small fraction into investment certificates when current volatility is low, and vice versa when it is high. The upper graph in the left column of Figure 3 shows the optimal positions in the stock, the discount certificate with a strike price of 90% and the money market account as a function of initial volatility. The position in the certificate increases almost linearly with volatility from around 25% to more than 70% for a volatility of about 16%, and stays rather constant for higher volatilities. The upper graph in the right column shows the corresponding factor exposures. While the exposures to jump risk and stock price diffusive risk are practically constant across volatilities, the exposure to volatility risk reaches its maximum in absolute terms for a volatility of 20%.

Despite the significant positions in certificates, the overall benefit from including retail derivatives in the portfolio is rather small. For a CRRA investor with $\gamma = 2$, the maximal additional certainty equivalent return $\mathcal{R}_{disc,w/o \to w}$ per year is 35 basis points. As the first additional benchmark, we consider the case where the investor has access to two plain vanilla options, which gives him one additional degree of freedom, and where he is allowed to short these options. The results are shown in Table 4. The maximal additional certainty equivalent return is now 39 bp. With one retail certificate only, the investor can already realize more than 90% of this utility gain. The second additional benchmark is the case of a complete market. The additional certainty equivalent return is more than

800 bp as compared to a static investment into the stock and the money market account only. Note, however, that the investor would have to trade continuously in infinitely many derivatives to achieve this utility gain.

The investors realize the highest utility improvement when they use discount certificates with a strike price equal to 90% of the current stock price. This contract is optimal not only for the most risk-averse investor, but also for investors with a quite low level of risk aversion. The other two discount certificates with strike prices equal to 100% and 110% of the current stock price perform only slightly worse, with utility improvements of 33 bp and 32 bp. Sprint certificates yield slightly worse results with an $\mathcal{R}_{disc,w/o \to w}$ of 30 bp.

Rolling discount certificates perform significantly worse than discount certificates. If the underlying discount certificates have a time to maturity of one month only, the maximal $\mathcal{R}_{disc,w/o \to w}$ is 11 bp. For a maturity of two years and a monthly roll-over, this number increases to 21 bp. Different from the case of discount certificates, the choice of the moneyness now plays an important role. While rolling discount certificates with a short maturity and with a strike of 100% yield an $\mathcal{R}_{disc,w/o \to w}$ of 11 bp, those with a strike of 97% yield 4 bp only. For a maturity of two years, the maximal improvement of 21 bp is achieved by setting the moneyness equal to 70%.

When we turn to path-dependent contracts like bonus certificates and turbos and to guarantee certificates, the picture changes completely. Bonus certificates with an initial time-to-maturity of two years, which are the best out of this group, yield an improvement of at most 13 bp, about one third of what can be achieved by discount certificates. Bonus certificates with a shorter time-to-maturity of two years yield slightly worse results with about 8 bp. The return characteristics of turbos are actually so bad that the investor would rather want to take a short position (which he cannot implement due to short-selling restrictions) than a long position. The same holds true for guarantee certificates.

5.2 Impact of Transaction Costs

Up to now, we have assumed that retail certificates can be bought and sold at their model prices. However, this is not necessarily the case. Certificates are not freely traded, but they can only be bought from and sold back to the issuer, so that deviations from model prices do not necessarily imply (exploitable) arbitrage opportunities. Empirical studies rather show that certificates tend to be overpriced at the beginning (when investors buy them) and underpriced towards the end (when investors sell them). We interpret these deviations more generally as transaction costs.

For the size of these deviations, Stoimenov and Wilkens (2005), e.g., estimate the regression model

$$\frac{p(CERT^i) - p(REP^i)}{p(REP^i)} = a + bL_i + \epsilon_i, \tag{6}$$

which relates the difference between the market prices $p(CERT)$ and the prices of replicating portfolios $p(REP)$ based on options traded at the EUREX to the remaining relative life-time L of the certificate. The estimates obtained for the constant a are 0.28% for discount certificates and 7.39% for sprint certificates (called turbos in their paper). The estimated slope coefficients b are -1.55% for discount certificates and -19.31% for sprint certificates.

We again use certificates with a time to maturity of two years (roughly equal to the average initial time to maturity in Stoimenov and Wilkens (2005)), buy them at initiation and hold them for one year. The analysis is done for discount certificates and sprint certificates, while we omit the other contracts for two reasons. First, the replicating portfolio using plain vanilla options traded at the EUREX can be set up independently of any model in the case of discount certificates and sprint certificates (as well as for rolling discount certificates), while it has to be based on some model assumptions for the other, path-dependent contracts. Second, discount and sprint certificates performed best without taking transaction costs into account, while the utility gains from the other contracts might not be high enough to compensate for transaction costs.

The results are given in Table 5. Discount certificates are the only retail derivatives from which the investor still benefits. For all other contracts, the transaction costs more than offset the utility gains from these contracts. However, even for discount certificates the additional certainty equivalent return drops from around 35 bp to at most 14.3 bp. The optimal strike price is still equal to 90% of the current stock price. The performance of the other two certificates is slightly worse.

For sprint certificates, the transaction costs are much higher than for discount certificates. As a consequence, these contracts, which were quite comparable to discount certificates in terms of utility improvement, now become so unattractive that the rational investor does not want to hold them anymore.

5.3 Interpretation

The benefits of an investment in certificates are low for several reasons. First, the investor does not rebalance his portfolio for one whole year. Branger, Breuer, and Schlag (2007) show that the rebalancing frequency is a major determinant of the utility gain that can be realized by investing in options, and that the investor should rebalance his portfolio at least monthly. Second, transaction costs are of the same order of magnitude as the utility gains, so that their inclusion makes an investment in certificates nearly worthless. We will now interpret the results and especially the differences between the various retail derivatives in more detail.

Table 4 gives the results for our three benchmark cases. In the ideal case of a complete market and with continuous trading, the investor has a positive exposure to stock diffusion risk, a negative exposure to volatility risk, and a negative exposure to jump risk. If he can only trade the stock and the money market account and is restricted to discrete trading, there is no exposure to volatility risk, a lower exposure to stock diffusion risk, and a slightly higher exposure to jump risk. With two derivatives, the investor would be able to match the initial exposures to the diffusion risks and to one jump size from the ideal case. However, the exposure to volatility risk and also to stock diffusion risk is significantly lower. Furthermore, the optimal initial exposure with discrete trading depends on the current level of volatility, and while the volatility risk exposure increases in absolute terms in volatility, the stock diffusion risk exposure and the jump risk exposure decrease.

When the investor can use only one retail certificate, he has two objectives. First, he tries to come as close as possible to the optimal initial exposure given by the ideal case in Table 4. He thus looks for derivatives that offer him a certain (optimal) relation between stock diffusion risk, volatility risk and jump risk. Second, he has to take into account that the exposure of his portfolio is going to change over time, and this foresight will have an impact onto his choice today. This trade-off between matching the initial exposures and stability over time is similar to the situation with monthly rebalancing and access to plain vanilla options as discussed in Branger, Breuer, and Schlag (2007).

As can be seen from Table 4, the investor wants a negative volatility risk exposure, which can be achieved by a long position in all certificates except short turbos and guarantee certificates. In the ideal case of a complete market, the optimal volatility risk exposure does not depend on the current volatility. Since the vega of all contracts decreases in absolute terms in volatility, the portfolio weight of all certificates should increase in volatility.

For discount certificates, the optimal position is in line with these arguments. The investor takes a long position in the certificates, which increases in volatility and provides a negative exposure to volatility risk. At the same time, it gives a negative exposure to downward jumps which turns out to be too high in absolute terms, and the investor faces a trade-off between a (desired) high volatility risk exposure and an (undesired) high jump risk exposure. As can be seen from Table 3, this results in a much less pronounced exposure to volatility risk and a slightly larger exposure to jump risk than in the ideal case and in the case with two standard calls, which are both given in Table 4. This trade-off also implies that the investor will prefer discount certificates for which the ratio of vega over jump risk exposure is rather large. The sensitivities are given in Table 2, and the largest ratio is indeed achieved for a strike price equal to 90% of the current stock price, the optimal choice.

The utility losses can be attributed to a suboptimal initial exposure and to the exposure not being stable over time. With one retail certificate instead of two options, the investor is restricted in terms of the initial exposure he can achieve. Given that the utility gain is only reduced from 39 bp to 35 bp, however, the differences in exposures are not too severe. The much more severe problem is the missing stability of the exposure over time, as can be seen from the much higher certainty equivalent excess return in case of a complete market with continuous trading.

Sprint certificates also offer a negative volatility risk exposure. In addition, they have a slightly higher ratio of vega to jump risk exposure than discount certificates, so that they might be even superior. Nevertheless, the utility gain is slightly smaller. The optimal initial exposures to volatility risk and jump risk and the portfolio weights are also smaller than for discount certificates.

The rather small utility improvement for bonus certificates with a time to maturity of one year can be attributed to two reasons. First, the initial exposures offered by this contract do not match the optimal profile. The negative volatility risk exposure is rather high for low volatilities, but decreases fast in absolute terms when volatility increases, while the jump risk exposure increases in absolute terms for an increase in volatility. This results in an extreme decrease in the ratio of vega to jump risk exposure when volatility increases, and the volatility exposure of the portfolio is not stable across volatility levels. There is yet a second reason why the exposures are rather unstable over time. The certificate is held until maturity, when e.g. vega goes to zero. Furthermore, when the barrier is touched, the bonus certificate changes into a simple stock with no volatility risk exposure at all.

For longer times to maturity of the bonus certificates, the performance becomes slightly better. One reason is that the optimal positions in the stock and the certificate change less when volatility increases. The exposures will also be more stable over time (at least if the lower threshold is not crossed), since the certificate is not held until maturity.

The bad performance of short turbos is not surprising. With a negative exposure to stock price diffusive risk, a positive exposure to volatility risk and a positive exposure to negative jumps, all three exposures have the wrong sign. Explaining the extremely bad performance of long turbos is more difficult. Like for bonus certificates, one reason is the extreme decrease in the ratio of vega to jump risk exposure when volatility increases. Furthermore, the exposures of long turbos are even more unstable over time. The value of the turbo is zero once it was knocked out, which dramatically changes the exposure of the portfolio.

Guarantee certificates have a positive exposure to volatility risk. Given that the volatility risk premium is negative, however, the investor wants a negative exposure to earn the premium. He would thus rather want to take a short position in guarantee certificates.

Rolling discount certificates based on discount certificates with one month to maturity also perform rather poorly. Compared to discount certificates with two years to maturity, they have a smaller exposure to volatility risk and a larger one to jump risk. The trade-off between vega and jump risk exposure is thus much worse. Furthermore, the embedded short call options are always held until maturity, when their vega will drop to zero and their stock price exposure will be either zero or one. Therefore, the exposures are rather unstable over time. As the small utility gains show, these problems can not be offset by re-setting the contract characteristics every month. However, these arguments also imply that a variant of rolling discount certificates where long-term discount certificates are rolled over every month might perform much better.

We therefore also considered rolling discount certificates based on discount certificates with two years to maturity which are rolled over every month. They perform indeed better than the usual rolling discount certificates. For the optimal moneyness of 70%, $\mathcal{R}_{disc,w/o\to w}$ increases to 21 bp[6]. However, this is still less than two thirds of the improvement realized when using standard discount certificates with two years to maturity.

5.4 Statistical Significance

To see whether our results are statistically significant, we use a t-test to assess whether the investor's indirect utility is significantly higher with retail derivatives than without. We do this for two representative cases, the discount certificate with a strike price equal to 90% of the stock price and the bonus certificate, the first representing a well-performing, the latter representing a not-so-well performing certificate. We obtain t-statistics of 48.6 in the first case and 24.3 in the second case, which both indicate significance at all conventional significance levels.

5.5 Robustness Checks

Our results up to now have shown that the investor mainly benefits from trading discount certificates, with sprint certificates coming in second. Retail derivatives with a barrier and certain knock-in or knock-out features perform much worse. Given the higher transactions costs for these contracts than for standard discount certificates, they will not be bought at all by the CRRA investor.

We redo the analysis for the case of loss aversion, similar to the empirical study on

[6]For higher risk aversions, certificates with lower moneyness perform slightly better. For $\gamma = 2$, however, a moneyness of 70% is best due to the short-selling constraint on the money market account.

plain vanilla options in Driessen and Maenhout (2007). Loss aversion utility is defined as:

$$U(W_T) = \begin{cases} (W_T - RL)^\alpha & \text{if } W_T \geq RL \\ \beta(RL - W_T)^\alpha & \text{if } W_T < RL \end{cases}.$$

We take $\alpha = 0.88$, as estimated by Tversky and Kahnemann (1992), and set β equal to the standard value of -2. As reference level RL, we use the initial wealth, i.e. $RL = 1.0$. To calculate the certainty equivalent wealth, the utility function needs to be inverted. Since the optimal expected utility will be larger than the utility of an investment in the money market account, which is already above the reference level, we invert the positive branch of the utility function.

The utility gain from discount certificates is equal to 62 bp, 53 bp, and 46 bp for strike prices of the embedded option equal to 90%, 100% and 110% of the stock price, respectively. Again, discount certificates with the lowest strike price perform best. The good performance of discount certificates can be explained by the lower exposure of these contracts to jump risk. This lowers the risk of ending up below the reference level, and the reduction of this probability increases the utility of a loss averse investor. With transaction costs, the utility gains drop to 11 bp, 8 bp, and 4 bp respectively. The utility loss due to transaction costs is thus larger than with CRRA utility.

The utility improvements for the other certificates are significantly smaller. Sprint certificates yield an improvement of 17.5 bp, while rolling discount certificates yield at most 8.6 bp when discount certificates with 2 years to maturity and a strike price of 70% of the current stock price are rolled over. Bonus certificates yield an $\mathcal{R}_{disc,w/o\rightarrow w}$ of only 9 bp. Taken together, loss aversion does not help to explain the high demand for retail derivatives. Again, discount certificates are the best choice for the investor, while he should not invest into more sophisticated contracts.

In case of loss aversion, the investor now also profits from guarantee certificates, which he did not want to invest in at all for CRRA utility. For a low volatility, he invests 52% of this wealth in the certificate and nothing in the stock. If volatility is high, he invests all his wealth in the stock and does not use guarantee certificates. The utility gain, however, is rather small and amounts to 4.3 bp only.

It is interesting to note that, while the qualitative results in terms of the utility improvement are comparable to the CRRA case, the optimal portfolio strategies are rather different. In case of loss aversion, the investor either holds the stock (for low volatility levels) or the discount certificates (for high volatility levels), but there is no gradual adjustment of the portfolio to an increasing volatility as for CRRA utility. If the discount certificate is replaced by the bonus certificate, then the investor neglects it for low volatilities, starts to replace the money market account by the certificate for an increasing volatility, and shifts partially back from bonus certificates to stocks for even higher volatilities.

6 Conclusion

An investor with CRRA utility benefits from having access to discount certificates and, to a slightly lower degree, sprint certificates. Retail derivatives with a more sophisticated payoff structure like rolling discount certificates, and in particular derivatives including barriers are much less attractive. Some are not bought by the investor at all, and for those he actually wants to trade, utility gains are significantly lower than for discount certificates. Taking realistic transaction costs into account reduces these gains even further. Put together, standard CRRA preferences cannot explain the huge demand for retail derivatives.

Further research could try to explain this demand by changing the assumptions on the preferences or the behavior of the investors. In terms of preferences, it might be interesting to consider more sophisticated concepts like Epstein-Zin utility or habit formation. Where the behavior of the investors is concerned, one promising approach might be to assume overconfidence and subjective beliefs. An investor who is convinced that the stock price will never hit a lower barrier, e.g., will have a high demand for bonus certificates or long turbos.

Another line of research could focus on contract design. The low utility gains from trading derivatives (and the higher utility gains under the assumption of continuous trading and market completeness) show that the retail derivatives considered do not bring the investor close enough to the overall optimal payoff structure. The challenge is then to find a contract specification which is at the same time simple and quite close to the overall optimal payoff.

A Computation of Optimal Portfolio

To calculate the optimal portfolio, we rely on a numerical approach. For each initial variance we consider, we simulate 500,000 paths of stock prices and volatilities. The initial prices of the derivative contracts are calculated by Fourier inversion or Monte Carlo simulation, depending on the type of contract. For the prices at which the derivatives can be sold at the end of the planning horizon (in our case after one year), we have to distinguish between path-dependent and path-independent claims. For the path-independent certificates, we tabulate the terminal prices on a two-dimensional grid of terminal stock prices and variances, and use linear interpolation later on. For the contracts with knock-in or knock-out features, the behavior of the stock price path is monitored. The price of the contract of then tabulated twice, both for the case that the barrier has been hit and for the case that the barrier has not been hit. Again, the price is either calculated by Monte-Carlo simulation or by Fourier inversion. The prices of the rolling discount certificates can be calculated along the paths using call prices obtained by Fourier inversion.

Given the initial and terminal prices for all paths, a numerical optimization routine is used to maximize the average utility of terminal wealth over the share of initial wealth invested in the stock and the share of initial wealth invested in the derivative (with the remainder being invested in the money market account) subject to short-selling constraints. This results in a maximal expected utility for every variance level. In a last step, we average over the different variance levels, where the weights are given by the distribution of the variance after one year when we start at the long-run mean, which is very close to the long-run distribution.

B Benchmark Case: Complete Market, Continuous Trading

We solve for the optimal indirect utility and the optimal exposures in case of continuous trading and market completeness. The setup is very similar to Liu and Pan (2003), with the exception that we consider a stochastic jump size in the stock price instead of a deterministic one.

The dynamics of the wealth W are given by

$$\begin{aligned} dW_t &= rW_t dt + \theta_t^{B1} W_t \left(\eta_1 V_t dt + \sqrt{V_t} dB_t^1\right) + \theta_t^{B2} W_t \left(\eta_2 V_t dt + \sqrt{V_t} dB_t^2\right) \\ &\quad + W_{t-} \left(\theta^N(X) dN_t - E_t^Q[\theta^N(X)] \lambda^{\mathbb{Q}} V_t dt\right), \end{aligned} \tag{7}$$

where θ_t^{B1} and θ_t^{B2} are the relative exposures to stock diffusion risk $\sqrt{V_t} dB_t^1$ and pure volatility diffusion risk $\sqrt{V_t} dB_t^2$, and where $\theta^N(X)$ is the percentage change in the investors wealth for a jump of size $e^X - 1$ in the stock.

The investor maximizes the utility from terminal wealth. The indirect utility function $J(t, w, v)$ is given by

$$J(t, w, v) = \max_{\left\{\theta_s^{B1}, \theta_s^{B2}, \theta_t^N(X), t \le s \le T\right\}} E\left[\frac{1}{1-\gamma} W_T^{1-\gamma} \middle| W_t = w, V_t = v \right],$$

subject to the wealth dynamics in Equation (7). The corresponding Hamilton-Jacobi-

Bellman (HJB) equation is

$$\max_{\{\theta_t^{B1},\theta_t^{B2},\theta_t^N\}} \Big\{ J_t + W_t J_W \left(r + \theta_t^{B1}\eta^{B1}V_t + \theta_t^{B2}\eta^{B2}V_t - E_t^{\mathbb{Q}}[\theta^N(X)]\lambda^{\mathbb{Q}}V_t\right)$$
$$+ \kappa^{\mathbb{P}}(\bar{v}^{\mathbb{P}} - V_t)J_V$$
$$+ \frac{1}{2}\, W_t^2 J_{WW} V_t \left[(\theta_t^{B1})^2 + (\theta_t^{B2})^2\right]$$
$$+ \frac{1}{2}\sigma_V^2 V_t J_{VV}$$
$$+ \sigma_V V_t W_t J_{WV}\left(\rho\theta_t^{B1} + \sqrt{1-\rho^2}\theta_t^{B2}\right)$$
$$+ \lambda^{\mathbb{P}} V_t E_t^{\mathbb{P}}\left[J(t, w(1+\theta^N(X)), v) - J(t,w,v)\right]\Big\} = 0, \tag{8}$$

where subscripts denote partial derivatives. We use the standard guess for the functional form of the indirect utility function

$$J(t,w,v) = \frac{w^{1-\gamma}}{1-\gamma}\exp\left\{\gamma h(\tau) + \gamma H(\tau)v\right\}, \tag{9}$$

where the function h and H depend on time to maturity only. Plugging this guess into the HJB-equation and taking partial derivatives gives the optimal exposures to the fundamental risk factors

$$\theta_t^{*B1} = \frac{\eta^{B1}}{\gamma} + \rho\sigma_V H(\tau) \tag{10}$$
$$\theta_t^{*B2} = \frac{\eta^{B2}}{\gamma} + \sqrt{1-\rho^2}\sigma_V H(\tau) \tag{11}$$
$$\theta_t^{*N}(X) = \left(\frac{\lambda^{\mathbb{P}}p(X)}{\lambda^{\mathbb{Q}}q(X)}\right)^{1/\gamma} - 1, \tag{12}$$

where $p(X)$ and $q(X)$ are the densities of the jump size under $\mathbb{P}$ and $\mathbb{Q}$, respectively. Plugging these exposures into the HJB-equation and sorting terms gives two ordinary differential equations for h and H:

$$h'(\tau) = \kappa^P\bar{v}^P H(\tau) + \frac{1-\gamma}{\gamma}\, r \tag{13}$$
$$H'(\tau) = a + c\, H(\tau) + d\, H^2(\tau) \tag{14}$$

with boundary conditions $h(0) = H(0) = 0$ and

$$\begin{aligned} a &= \frac{1-\gamma}{2\gamma^2}\left[(\eta^{B1})^2 + (\eta^{B2})^2\right] + \frac{1-\gamma}{\gamma}\,\lambda^Q - \frac{1}{\gamma}\,\lambda^P + \lambda^Q E_t^{\mathbb{Q}}\left[\left(\frac{\lambda^{\mathbb{P}} p(X)}{\lambda^{\mathbb{Q}} q(X)}\right)^{1/\gamma}\right] \\ c &= -\kappa^P + \frac{1-\gamma}{\gamma}\sigma_V\left(\rho\eta^{B1} + \sqrt{1-\rho^2}\eta^{B2}\right) \\ d &= \frac{1}{2}\,\sigma_V^2. \end{aligned}$$

The ordinary differential equation for H is a standard Ricatti equation which can be solved in closed form. The function h then follows by integration.

References

Bakshi, G., and N. Kapadia, 2003, Delta-Hedged Gains and the Negative Market Volatility Risk Premium, *Review of Financial Studies* 16, 527–566.

Bates, D. S., 2000, Post-'87 Crash Fears in the S&P Futures Option Market, *Journal of Econometrics* 94, 181–238.

Branger, N., B. Breuer, and C. Schlag, 2007, Optimal Derivative Strategies with Discrete Rebalancing, *Working Paper*.

Branger, N., C. Schlag, and E. Schneider, 2008, Optimal Portfolios When Volatility can Jump, *Journal of Banking and Finance* 32, 1087–1097.

Breuer, W., and A. Perst, 2006, Retail Banking and Behavioral Financial Engeneering: The Case of Structured Products, *Journal of Banking and Finance* 31, 827–844.

Broadie, M., M. Chernov, and M. Johannes, 2008, Understanding Index Option Returns, *Working Paper*.

Bundesverband Deutscher Banken, 2007, Deutsche investieren immer weniger in Aktien - Mehr als 4,5 Billionen Euro Geldvermögen, http://www.bankenverband.de/channel/111010/art/2120/index.html.

Coval, J., and T. Shumway, 2001, Expected Option Returns, *Journal of Finance* 56, 983–1009.

Deutsches Derivate Institut, 2007, Branchenverband DDI zieht Rekordbilanz für deutschen Derivatemarkt 2006, http://gfx.finanztreff.de/ddi_proxy/img_cms/PM_22_01_07_DDI_Jahresstatistik2006.pdf.

Driessen, J., and P. Maenhout, 2007, A Portfolio Perspective on Option Pricing Anomalies, *Review of Finance* 11, 561–603.

Duffie, D., J. Pan, and K. Singleton, 2000, Transform Analysis and Asset Pricing for Affine Jump Diffusions, *Econometrica* 68, 1343–1376.

Eraker, B., 2004, Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices, *Journal of Finance* 59, 1367–1403.

Guse, F., and M. Muck, 2006, Jump Risk Premia Implicit in DAX Options - A Note on Implied State-GMM Estimation of Stochastic Volatility Jump Diffusion Models, *Working Paper.*

Jones, C., 2006, A Nonlinear Factor Analysis of S&P 500 Index Option Returns, *Journal of Finance* 61, 2325–2363.

Liu, J., and J. Pan, 2003, Dynamic Derivative Strategies, *Journal of Financial Economics* 69, 401–430.

Muck, M., 2006, Where Should You Buy Your Options? The Pricing of Exchange Traded Certificates and OTC-Derivatives in Germany, *Journal of Derivatives* 14, 82–96.

Muck, M., 2007, Pricing Turbo Certificates in the Presence of Stochastic Jumps, Interest Rates, and Volatility, *Die Betriebswirtschaft* 67, 224–240.

Pan, Jun, 2002, The Jump-Risk Premia Implicit in Options: Evidence form an Integrated Time-Series Study, *Journal of Financial Economics* 8, 232–361.

Stoimenov, P., and S. Wilkens, 2005, Are structured products 'fairly' priced? An analysis of the German market for equity-linked instruments, *Journal of Banking and Finance* 29, 2971–2993.

Tauchen, G., and H. Zhou, 2006, Realized Jumps on Financial Markets and Predicting Credit Spreads, *Working Paper*.

Tversky, A., and D. Kahnemann, 1992, Advances in Prospect Theory: Cumulative Representation of Uncertainty, *Journal of Risk and Uncertainty* 5, 297–323.

Wilkens, S., C. Erner, and K. Röder, 2003, The Pricing of Structured Products - An Empirical Investigation of the German Market, *Journal of Derivatives* 11, 55–69.

Wilkens, S., and P. Stoimenov, 2007, The pricing of leverage products: An empirical investigation of the German market for 'long' and 'short' stock index certificates, *Journal of Banking and Finance* 31, 735–750.

κ^P	1.8	κ^Q	1.4
$\bar{v}^P$	0.028	$\bar{v}^Q$	0.036
σ_V	0.47		
ρ	-0.35		
μ_X^P	-1.8%	μ_X^Q	-2.7%
σ_X	11.0%		
λ	123.7		
η_1	0.85		
η_2	-0.59		
η_V	-0.4		

Table 1: $\mathbb{P}$- and $\mathbb{Q}$-Parameters and Risk Premia for the Option Pricing Model

The parameters of the Bates (2000) model with one stochastic volatility are calibrated using data on the DAX and DAX index options over the time period from January 1997 until December 2004. The overall annual equity premium is 5.5%, estimated based on data from January 1975 until December 2006.

Strike	Delta	Vega	$\Delta_{95\%}$	$\Delta_{80\%}$	$\Delta_{20\%}$	$\Delta_{5\%}$	$\frac{Vega}{\Delta_{5\%}}$
Stock							
	1.00	0.00	0.18	0.06	-0.09	-0.18	0.00
Discount Certificate - Maturity: 2 years							
100%	0.26/0.29	-0.78/-0.49	0.03/0.04	0.01/0.02	-0.03/-0.03	-0.07/-0.06	11.14/8.17
90%	0.17/0.23	-0.63/-0.40	0.02/0.03	0.01/0.01	-0.02/-0.02	-0.05/-0.05	12.60/8.00
110%	0.38/0.34	-0.89/-0.48	0.05/0.05	0.02/0.02	-0.04/-0.03	-0.10/-0.07	8.90/6.86
Sprint Certificate - Returns between 0% and 7% are doubled							
	0.43/0.37	-0.94/-0.50	0.09/0.08	0.06/0.05	-0.01/-0.01	-0.07/-0.05	13.43/10.00
Guarantee Certificate							
	0.46/0.25	0.66/0.19	0.09/0.05	0.03/0.02	-0.04/-0.02	-0.06/-0.04	-11.00/-4.75
Bonus Certificate - Knock-In Barrier 65%, Bonus Level 120%, Maturity 1 year							
	0.19/0.46	-2.52/-0.63	0.02/0.07	0.01/0.03	-0.02/-0.05	-0.07/-0.11	36.00/5.73
Bonus Certificate - Knock-In Barrier 65%, Bonus Level 120%, Maturity 2 years							
	0.30/0.37	-1.63/-0.50	0.04/0.06	0.02/0.02	-0.03/-0.04	-0.09/-0.08	18.11/6.25
Long Turbo - Strike: 90%, Barrier: 90%, Maturity 1 year							
	1.00/1.05	-1.54/0.03	0.18/0.18	0.06/0.06	-0.11/-0.11	-0.15/-0.14	10.27/-0.21
Short Turbo - Strike: 110%, Barrier: 110%, Maturity 1 year							
	-0.78/-0.87	1.44/0.01	-0.07/-0.10	-0.04/-0.05	0.07/0.07	0.16/0.15	9.00/0.07
Rolling Discount Certificate - Maturity 1 month							
97%	0.06/0.36	-0.55/-0.20	0.00/-0.02	0.00/-0.00	-0.05/-0.04	-0.14/-0.10	3.87/1.93
100%	0.32/0.43	-1.06/-0.21	0.01/0.05	0.01/0.02	-0.08/-0.05	-0.17/-0.11	6.24/1.91
Rolling Discount Certificate - Maturity 2 years							
70%	0.06/0.13	-0.32/-0.28	0.01/0.02	0.00/0.01	-0.01/-0.01	-0.02/-0.03	16.00/9.33

Table 2: Sensitivities of Certificates

The table shows the delta, the vega, and the price change in case of a jump corresponding to the x percentile of the jump size distribution. The last column gives the relationship between vega and the exposure to large negative jumps. The first value in each column is for an initial variance equal to the lowest variance level 0.003 (which corresponds to a annualized volatility of 5.5%), the second value in each column is for an initial variance equal to the highest variance level 0.19 (volatility = 43.6%).

Risk Aversion	$(\phi^\star)_0$	$(\psi^\star)_0$	θ^{B1}	θ^{B2}	$\theta^{N}_{5\%}$	$\mathcal{R}_{w/o\to w}$
Discount Certificate - ATM						
2	0.25/0.00	0.31/0.76	0.40/0.38	-0.13/-0.21	-0.07/-0.07	0.333%
4	0.12/0.00	0.18/0.41	0.20/0.21	-0.08/-0.11	-0.04/-0.04	0.189%
8	0.06/0.00	0.10/0.21	0.10/0.10	-0.04/-0.06	-0.02/-0.02	0.099%
Discount Certificate - ITM (Strike: 90% of Initial Stock Price)						
2	0.26/0.00	0.37/0.81	0.39/0.36	-0.13/-0.22	-0.07/-0.06	0.352%
4	0.12/0.00	0.22/0.44	0.20/0.19	-0.08/-0.12	-0.04/-0.03	0.202%
8	0.06/0.00	0.12/0.23	0.10/0.10	-0.04/-0.06	-0.02/-0.02	0.106%
Discount Certificate - OTM (Strike: 110% of Initial Stock Price)						
2	0.25/0.00	0.27/0.71	0.41/0.41	-0.12/-0.21	-0.10/-0.08	0.315%
4	0.11/0.00	0.16/0.38	0.21/0.21	-0.07/-0.11	-0.04/-0.04	0.177%
8	0.05/0.00	0.09/0.19	0.11/0.11	-0.04/-0.05	-0.02/-0.02	0.093%
Sprint Certificate - Returns between 0 and 7% are Doubled						
2	0.25/0.00	0.24/0.70	0.41/0.41	-0.11/-0.20	-0.06/-0.05	0.303%
4	0.12/0.00	0.14/0.37	0.21/0.22	-0.07/-0.10	-0.03/-0.02	0.170%
8	0.06/0.00	0.08/0.19	0.11/0.11	-0.04/-0.05	-0.02 /-0.01	0.089%
Guarantee Certificate						
no investment in certificate						
Bonus Certificate - Knock-In Barrier 65%, Bonus Level 120%, Maturity 1 year						
2	0.33/0.25	0.18/0.25	0.43/0.41	-0.21/-0.09	-0.07/-0.07	0.083%
4	0.16/0.11	0.11/0.15	0.22/0.21	-0.12/-0.05	-0.04/-0.04	0.056%
8	0.08/0.05	0.06/0.08	0.11/0.10	-0.07/-0.03	-0.02/-0.02	0.032%
Bonus Certificate - Knock-In Barrier 65%, Bonus Level 120%, Maturity 2 years						
2	0.24/0.22	0.31/0.30	0.41/0.39	-0.22/-0.08	-0.07/-0.07	0.130%
4	0.11/0.10	0.18/0.18	0.21/0.20	-0.13/-0.05	-0.04/-0.04	0.085%
8	0.06/0.05	0.09/0.10	0.11/0.10	-0.07/-0.03	-0.02/-0.02	0.048%
Long Turbo - Strike: 90 %, Barrier: 90 %, Maturity 1 and 2 years						
no investment in turbo						
Short Turbo - Strike: 110 %, Barrier: 110%, Maturity 1 and 2 years						
no investment in turbo						
Rolling Discount Certificate - Maturity 1 month, Initial Strike: 100% of Stock Price						
2	0.08/0.39	0.55/0.00	0.36/0.39	-0.26/0.00	-0.11/-0.07	0.112%
4	0.04/0.19	0.29/0.00	0.18/0.19	-0.14/0.00	-0.06/-0.03	0.062%
8	0.02/0.09	0.15/0.00	0.09/0.09	-0.07/0.00	-0.03/-0.02	0.033%
Rolling Discount Certificate - Maturity 1 month, Initial Strike: 97% of Stock Price						
2	0.24/0.39	0.35/0.00	0.29/0.39	-0.09/0.00	-0.09/-0.07	0.042%
4	0.12/0.19	0.19/0.00	0.15/0.19	-0.05/0.00	-0.05/-0.03	0.024%
8	0.06/0.09	0.10/0.00	0.07/0.09	-0.03/0.003	-0.03/-0.02	0.014%
Rolling Discount Certificate - Maturity 2 years, Initial Strike: 70% of Stock Price						
2	0.26/0.15	0.67/0.85	0.34/0.30	-0.09/-0.11	-0.06/-0.05	0.210%
4	0.12/0.01	0.39/0.72	0.17/0.14	-0.05/-0.09	-0.03/-0.02	0.138%
8	0.06/0.00	0.21/0.39	0.08/0.07	-0.03/-0.05	-0.01/-0.01	0.076%

Table 3: Optimal Investment with Certificates, Investment Horizon: 1 year

The certificates have an initial maturity of 2 years, expect for the rolling discount certificate which is perpetual and the bonus certificate which has a maturity of 1 or 2 years. For rolling discount certificates, 'maturity' is the initial maturity of the discount certificates that are rolled over every months. γ is the risk aversion of the investor, $(\phi^\star)_0$ is the optimal position in the stock, $(\psi^\star)_0$ is the optimal position in the certificate at time 0. The θ's give the corresponding exposures to the risk factors. The first value in each column is for an initial variance equal to the lowest variance level 0.003 (which corresponds to a annualized volatility of 5.5%), the second value in each column is for an initial variance equal to the highest variance level 0.19 (volatility = 43.6%). $\mathcal{R}_{w/o\to w}$ is the additional certainty equivalent return due to derivatives.

γ	$(\phi^\star)_0$	$(\psi_1^\star)_0$	$(\psi_2^\star)_0$	θ^{B1}	θ^{B2}	$\theta^N_{5\%}$	$\mathcal{R}_{w/o\to w}$
			Stock and MMA only				
2	0.40/0.39			0.40/0.39	0.00/0.00	-0.07/-0.07	0.0%
4	0.20/0.19			0.20/0.19	0.00/0.00	-0.04/-0.03	0.0%
8	0.10/0.09			0.10/0.09	0.00/0.00	-0.02/-0.02	0.0 %
			Stock, MMA, 110% call, 100% call				
2	0.81/1.21	0.18/-0.21	-0.33/0.83	0.35/0.24	-0.08/-0.47	-0.06/-0.04	0.383%
4	0.44/0.68	0.10/-0.57	-0.18/0.36	0.18/0.13	-0.06/-0.25	-0.03/-0.02	0.218%
8	0.23/0.35	0.05/-0.28	-0.09/0.17	0.09/0.07	-0.03/-0.13	-0.02/-0.01	0.115%
			Stock, MMA, 90% call, 100% call				
2	1.03/1.33	-0.57/0.54	0.31/-1.01	0.35/0.23	-0.07/-0.41	-0.06/-0.04	0.388%
4	0.57/0.75	-0.31/0.22	0.16/-0.49	0.18/0.13	-0.05/-0.22	-0.03/-0.02	0.223%
8	0.30/0.39	-0.16/0.11	0.08/-0.25	0.09/0.07	-0.03/-0.11	-0.02/-0.01	0.118%
			Complete Market				
2				0.55/0.55	-0.64/-0.64	-0.06/-0.06	8.161%
4				0.28/0.28	-0.34/-0.34	-0.03/-0.03	2.914%
8				0.15/0.15	-0.18/-0.18	-0.01/-0.01	1.297%

Table 4: Benchmark Cases for Optimal Investment, Investment Horizon: 1 year

The options have an initial maturity of 2 years. γ is the risk aversion of the investor, $(\phi^\star)_0$ is the optimal position in the stock, $(\psi_i^\star)_0$ is the optimal position in the ith derivative at time 0. The θ's give the corresponding exposures to the risk factors. The first value in each column is for an initial variance equal to the lowest variance level 0.003 (which corresponds to a annualized volatility of 5.5%), the second value in each column is for an initial variance equal to the highest variance level 0.19 (volatility = 43.6%). $\mathcal{R}_{w/o\to w}$ gives the increase in certainty equivalent wealth compared to a pure stock and money market account investment. $\mathcal{R}_{w/o\to w}$ is the additional certainty equivalent return due to derivatives.

Risk Aversion	$(\phi^\star)_0$	$(\psi^\star)_0$	θ^{B1}	θ^{B2}	$\theta^{N}_{5\%}$	$\mathcal{R}_{w/o\rightarrow w}$
Discount Certificate - ATM						
2	0.40/0.00	0.00/0.74	0.40/0.37	0.00/-0.20	-0.07/-0.06	0.138%
4	0.20/0.00	0.00/0.39	0.20/0.20	0.00/-0.11	-0.04/-0.03	0.077%
8	0.10/0.00	0.00/0.20	0.10/0.10	0.00/-0.06	-0.02/-0.02	0.040%
Discount Certificate - ITM (Strike: 90% of Initial Stock Price)						
2	0.40/0.00	0.00/0.79	0.40/0.35	0.00/-0.21	-0.07/-0.06	0.143%
4	0.20/0.00	0.00/0.42	0.20/0.19	0.00/-0.11	-0.04/-0.03	0.079%
8	0.10/0.00	0.00/0.22	0.10/0.10	0.00/-0.06	-0.02/-0.02	0.041%
Discount Certificate - OTM (Strike: 110% of Initial Stock Price)						
2	0.40/0.00	0.00/0.70	0.40/0.39	0.00/-0.20	-0.07/-0.07	0.132%
4	0.20/0.00	0.00/0.36	0.20/0.21	0.00/-0.10	-0.04/-0.04	0.073%
8	0.10/0.00	0.00/0.18	0.10/0.10	0.00/-0.05	-0.02/-0.02	0.038%
Sprint Certificate - Returns between 0 and 7% are Doubled						
no investment						

Table 5: Optimal Investment with Certificates under Consideration of Transaction Costs

The certificates have an initial maturity of 2 years. γ is the risk aversion of the investor, $(\phi^\star)_0$ is the optimal position in the stock, $(\psi^\star)_0$ is the optimal position in the certificate at time 0. The θ's give the corresponding exposures to the risk factors. The first value in each column is for an initial variance equal to the lowest variance level 0.003 (which corresponds to a annualized volatility of 5.5%), the second value in each column is for an initial variance equal to the highest variance level 0.19 (volatility = 43.6%). $\mathcal{R}_{w/o\rightarrow w}$ is the additional certainty equivalent return due to derivatives.

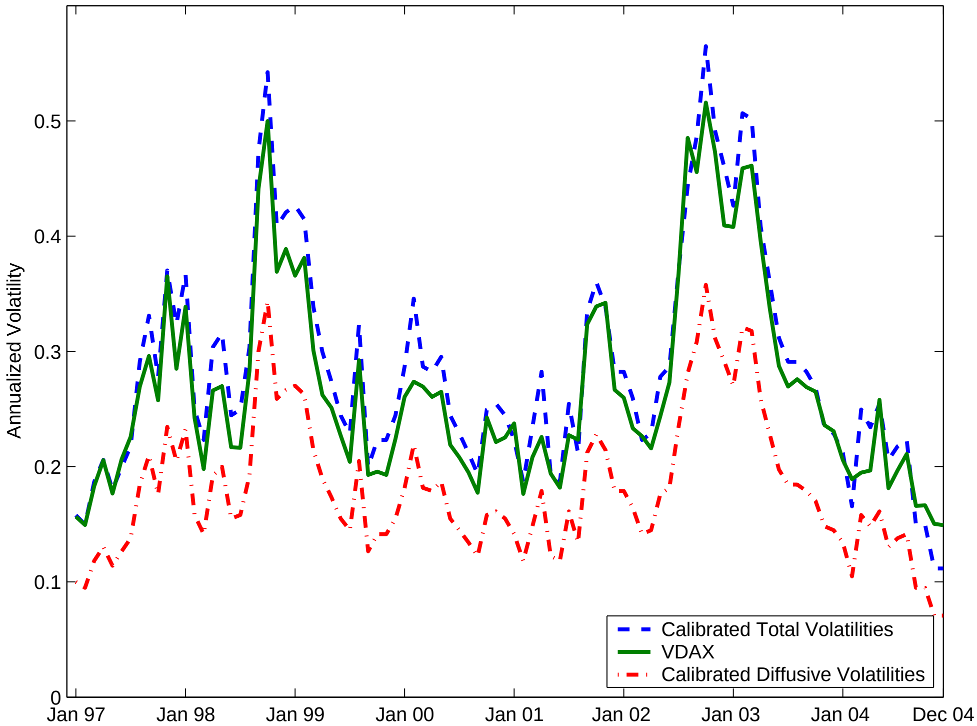


Figure 1: Calibrated Diffusive and Total Volatilities versus the VDAX
The figure shows the time series of the volatilities of the German DAX index over the time period from January 1997 until December 2004. The solid line is the VDAX (old version), the dash-dotted line is the local diffusive variance which results from the calibration to option prices. The dashed line is the local total variance, calculated as given inside the expectation in Equation (5).

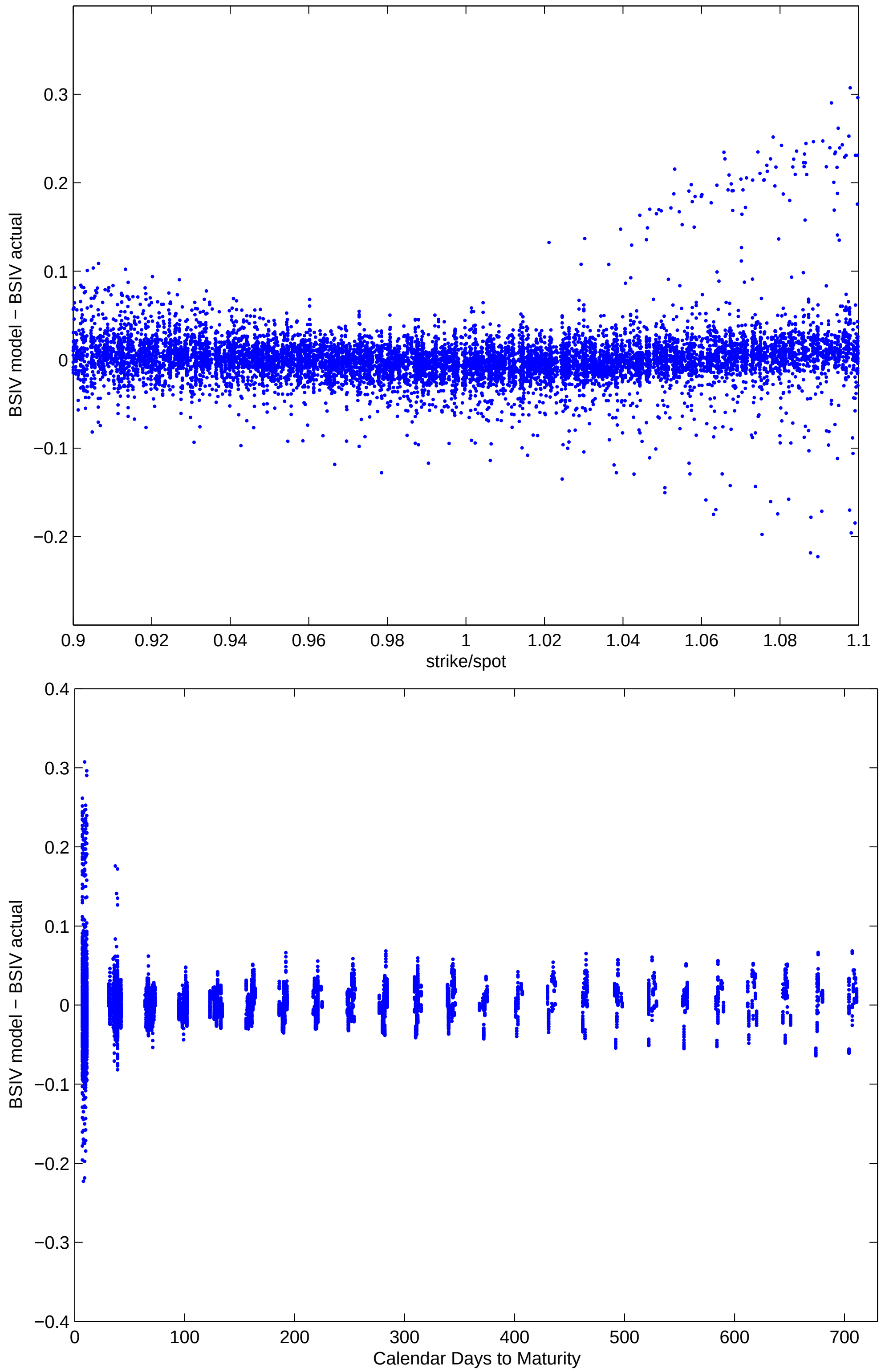
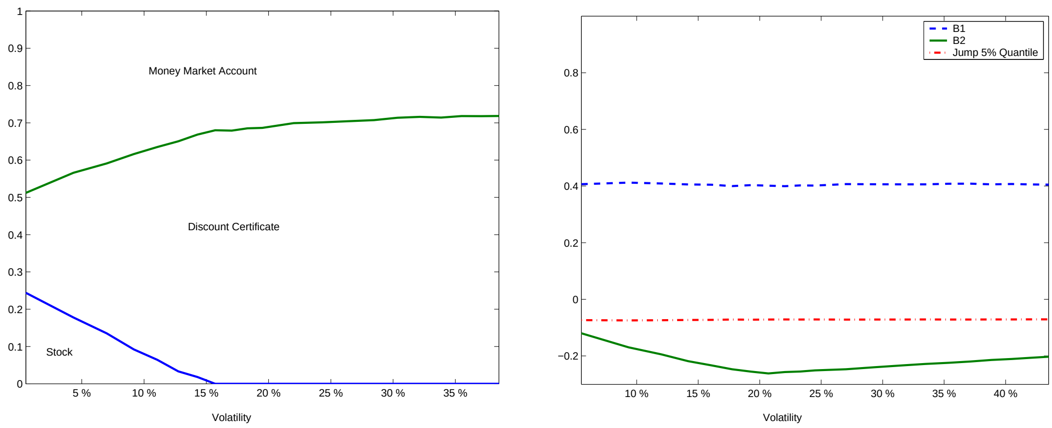


Figure 2: Differences in Black-Scholes Implied Volatilities

The figures show the difference between Black-Scholes implied volatilities of model and market prices for all (put) options used in the calibration as a function of moneyness (upper graph) and time to maturity (lower graph).

Stock and Discount Certificate with Model Pricing



Stock and Discount Certificate with Transaction Costs

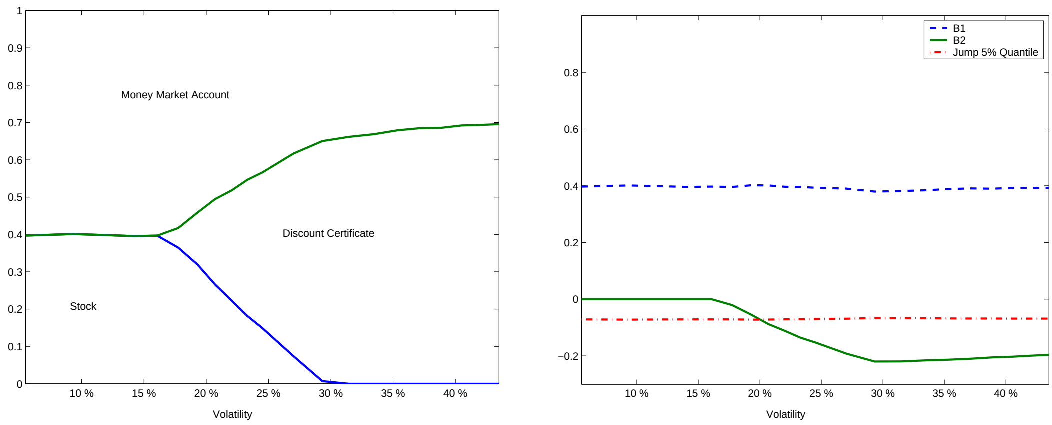


Figure 3: Optimal Initial Positions and Initial Exposures to Risk Factors

The figures show the optimal positions for the discount certificate with a strike price of 90% (left panel) and the resulting initial exposures to the risk factors (right panel). The investor has access to discount certificates with a strike price equal to 90% of the current stock price. In the upper row, the prices of the certificates are equal to the model prices. In the lower row, we also take transaction costs into account.