The dark side of the moon: structured products from the customer's perspective

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Abstract

The paper develops a framework for the design of optimal structured products allowing us to analyze the maximal utility gain for an investor that can be achieved by introducing structured products. We demonstrate on data from two of the largest markets for structured products (Germany and Switzerland) that most of the successful structured products are not optimal for a perfectly rational investor and we investigate the reasons that make them nevertheless look so attractive for many investors. Finally, using a large-scale field experiment, in which subjects freely designed structured products, we deduce how various factors effect the preferences on structured products.

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1 Importance of structured financial products

Structured products, SPs, combine one or more classical assets (stocks, bonds, indices) with at least one derivative into a bundle that shall have specific

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interesting features for investors, like capital protection or increased participation. They enable investors with comparatively low budget and knowledge to invest indirectly into derivatives. Banks profit from the restricted participation of these non-institutional investors in the derivatives market, in that they retain a margin profit when issuing structured products. Structured products are immensely popular in Europe. However, some European states like Norway recently introduced high regulatory burdens on them. Our paper contributes to these regulatory issues by analyzing the risks and benefits of structured products for private investors. Structured products are not yet as popular in the USA. Potential reasons for this are regulatory reasons, but also the fact that more private investors than in Europe hold stocks¹.

Note that the existence of structured products is puzzling since in traditional portfolio models there is no role for SPs. The classical mean-variance model of Markowitz [21], for example, suggests that an investment in the market portfolio and the risk-free asset would be sufficient to construct optimal investments for all degrees of risk-aversion – this is the famous Two-Fund-Separation, 2FS. Hence the only role for banks would be to offer the market portfolio at minimal cost, e.g. in the form of an exchange traded fund (ETF). Not only the widespread existence of structured products is puzzling, but also the large variety of payoff patterns that can be found. Even very complicated payoff patterns are often not as exotic and rare as one would expect: this can be seen in data from the German and the Swiss market (the two largest markets for structured products in Europe). The Swiss sample consists of N = 47'362 products issued from March 2007 to November 2007. The German sample consists of all structured products on the market (N = 270'254)as of end of November 2007. Figure 1 and Figure 2 show typical payoff diagrams for the most popular types in each country.² We observe that in each country a large variety of products are popular: leverage, capital protection and outperformance can all be found. At the same time there are clear similarities between Germany and Switzerland: three product types (discount certificates, bonus certificates and leverage products) can be found in the top list of both countries.

Structured products have so far been studied in academic research nearly exclusively from the issuer's perspective, mostly in the context of option pricing and hedging. (For an early exception see the article by Shefrin and Statman [27].) Indeed, building on the seminal paper of Black and Scholes [3] a new field of finance, called financial engineering, has emerged in which

¹Compare [2] for more on the regulatory background for structured products in the US.

²For clarification, we omit plain vanilla type put and call options that are occasionally also listed as structured products.



Figure 1: The most popular structured products in Germany with their respective shares of all issued products (N = 270254).

mathematicians and engineers developed more and more elaborate pricing techniques for ever more complicated structured products. Besides this huge technical literature, recently a few empirical studies on the actual market prices [32, 15, 31] of SPs can be found and there are interesting first explorative studies on specific puzzles regarding the investors' preferences for certain product classes [28, 16, 6, 7]. But all in all the investor's perspective on structured products is still uncharted territory, somehow "the dark side of the moon".

In this article we try to shed some light on this "dark side" and take the investor's perspective as the starting point for our expedition. In this way we ask whether structured products are an appropriate tool to improve investment performance and what types of products are optimal under normative, but also under behavioral models. In particular, we measure how big the potential improvement of a portfolio can be when adding structured products. We also demonstrate that the most popular products derive their popularity not from rational, but from behavioral factors like framing, loss aversion and



Figure 2: The same list for Switzerland (N = 47362).

probability mis-estimation.

The paper is organized as follows:

In the next section we provide a canonical model that we then use to design optimal structured products. Thereafter we reverse the point of view and ask for any given structured product which risk-preferences and biases could justify its existence. Finally we report results from a large field experiment in which participants could design structured products using a multi-touch table.

2 Designing optimal SPs

In the following we introduce a simple two-period model³ for a structured product, assuming that the market prices can be described by the CAPM or the Black-Scholes model.⁴ Using results on the co-monotonicity of optimal

³The majority of investors keeps their structured products until maturity.

⁴Many of our results could be obtained under much weaker assumptions, e.g. monotonicity of the likelihood ratio. We point out when this generalization is possible.

2.1 A canonical model

investments [23] and extending previous results on optimal investments in the strictly concave expected utility setting [20], we obtain qualitative properties that optimal structured products should satisfy under more and more relaxed conditions on the rationality of the investors. We will use these results to show later that the attractiveness of the currently most popular classes of SPs cannot be understood within a rational decision model.

Some of our analytical results are similar to independent work by Prigent [22] on portfolio optimization and rank-dependent expected utility. In particular, Prigent derives a solution for a basic optimization problem in the case of non-concave utility functions (see Lemma 1).

2.1 A canonical model

We say that a structured product is *optimal* if its payoff distribution maximizes the given utility of an investor under the constraint that the arbitragefree price of the product cannot exceed a certain value. It seems appropriate to use a two-period model, since most structured products are sold over the counter and have a fixed maturity. To obtain intuitive results, we will assume most of the time that the market can be described by the capital asset pricing model. Note that the validity of the CAPM is not at odds with violations of the two-fund separation property. For example heterogeneous beliefs or uncorrelated optimization mistakes give rise to heterogeneous portfolios while on the level of the market the security market line property of the CAPM can still hold [8, 13, 4]. Moreover, we assume that the market is complete for the product designer and that the investor does not hold other assets than the structured product (or at least considers them as a separate "mental account").⁵</sup>

It has been shown in [23] that any optimal structured product can be described by its return distribution at maturity as a function of the return of the market portfolio. Therefore we naturally arrive at the following utility maximization problem:

$$\max_{y \in \mathbb{R}^S} \sum_{s=1}^S p_s u(y_s),$$

subject to $\sum_s \pi_s y_s = B,$

where $u: \mathbb{R} \to \mathbb{R}$ is the utility function, B > 0 is the budget, $s = 1, \ldots, S$ mark the states that occur with probability $p_s > 0$ and $\pi = (\pi_1, \ldots, \pi_S) \gg 0$

⁵Most of our results carry over if the investor holds additionally risk-free assets.

are the state prices. y_s denotes the payoff of the structured product at maturity in state s.

It will turn out to be useful to transform this model slightly: normalize π_s to π_s^* with $\sum_s \pi_s^* = 1$, set B = 1, then we can define the likelihood ratio $\ell_s = \pi_s^*/p_s$. Absence of arbitrage implies $\sum_s \pi_s^* R_s^k = R$, where R_s^k is the return of asset k in state s and R is the risk-free rate. Therefore $\sum_s \pi_s = R$ and we can reformulate the optimization problem using the likelihood ratio process

$$\max_{y \in \mathbb{R}^{S}} \sum_{s=1}^{S} p_{s} u(y_{s}),$$

subject to $\sum_{s} p_{s} \ell_{s} y_{s} = R.$ (1)

While studying a discrete state space helps to find an intuition for this problem, it will turn out to be easier if we study a continuous state space. Particularly, we will not face the problem that we have to split states when studying non-concave utility functions. Therefore we continue with the continuous variant of (1) and maximize

$$U(y) := \int_{\mathbb{R}} u(y(x)) \, dp(x),$$

over all $y \in L^1(\mathbb{R}, \ell p)$, subject to the condition

$$\int_{\mathbb{R}} y(x)\ell(x)dp(x) = R$$

The likelihood ratio ℓ can be written in the CAPM as

$$\ell(x) = a - bx,$$

for some positive constants a,b, as has been shown in [5]. From the linearity of the likelihood ratio ℓ the security-market line of the CAPM can be derived by elementary computations.

In the case of the Black-Scholes model this can be written as $\ell(x) = Ae^{-Bx}$ for suitable A, B > 0.

2.2 Optimal SPs in the strictly concave case: convex payoffs

Let us assume that the investor has rational preferences, i.e. he follows the expected utility approach by von Neumann and Morgenstern [30]. The case

where the utility function u is strictly increasing and strictly concave has been studied previously by Kramkov and Schachermeyer [20]. In this case, a straightforward variational approach leads to the optimality condition

$$y(x) = v^{-1}(\lambda \ell(x)), \tag{2}$$

where v := u' and λ is a Lagrange parameter that has to be chosen such that (2) holds.

Note that unless the utility function is quadratic, in the two-period model the 2FS property does not hold for any distribution of returns⁶.

What conditions on y can we derive from (2) in general? In fact, in the most realistic case (when the investor is prudent⁷), y is convex, as the following theorem shows:

Theorem 1. Let u be a strictly increasing and strictly concave utility function. Assume furthermore that u'' > 0 (i.e. the investor is prudent). Then the return of an optimal SP on a CAPM market is strictly convex as a function of the market return. If u''' = 0 (i.e. u is quadratic) then the payoff function will be linear. If u''' < 0 the payoff function will be strictly concave.

Proof. Take the second derivative of $y(x) = v^{-1}(\lambda(a + bx))$ with v := u'. This gives:

$$y''(x) = -\frac{\lambda^2 b^2 v''(v^{-1}(\lambda(a+bx)))}{(v'(v^{-1}(\lambda(a+bx))))^3}.$$

The denominator is positive, since v' = u'' < 0 (*u* is strictly concave). Thus y'' is positive if u''' = v'' > 0.

This result is surprising since prudent investors prefer SPs that look similar to call options which are usually associated with gambling or risk-taking! In particular, the theorem shows that if u is quadratic, the return is a linear function of the market return, i.e. there is no need for a SP at all, which is the classical two-fund separation theorem of the mean-variance portfolio theory.

The following corollary gives a simpler condition that is well-known to finance under which strict convexity of y holds:

Corollary 1. If the investor has non-increasing absolute risk-aversion, then the payoff function should be strictly convex.

⁶In models with *time continuous* trading, *no transaction costs* and an underlying process that follows a *geometric Brownian motion* the 2FS property would hold for any class of utility functions with constant relative risk aversion.

⁷The importance of prudence is well known from the literature in insurance theory, see for example [14]

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Proof. Consider the Arrow-Pratt risk measure r(x) := -u''(x)/u'(x). Then

$$r'(x) = -\frac{u'''(x)u'(x) - (u''(x))^2}{(u'(x))^2}.$$
(3)

Hence, if $r' \ge 0$ (i.e. r is not increasing) then $u'''(x) \ge \frac{(u''(x))^2}{u'(x)}$. Since u is strictly increasing, this is positive. Thus v'' = u''' > 0 and we can apply Theorem 1.

Corollary 1 essentially also holds when we replace the CAPM with the Black-Scholes model:

Theorem 2. If the pricing is described by Black-Scholes, then the optimal payoff function is strictly convex if the investor has decreasing absolute risk aversion (e.g. constant relative risk aversion).

Proof. We compute the second derivative of y which becomes

$$y''(x) = \frac{\lambda A B^2 e^{-Bx}}{(v'(v^{-1}(\lambda A e^{-Bx})))} - \frac{\lambda^2 A^2 B^2 e^{-2Bx} v''(v^{-1}(\lambda A e^{-Bx}))}{(v'(v^{-1}(\lambda A e^{-Bx})))^3}.$$

This is positive if the following condition holds:

$$\lambda A e^{-Bx} v''(v^{-1}(\lambda A e^{-Bx})) - (v'(v^{-1}(\lambda A e^{-Bx})))^2 > 0.$$
(4)

Noticing that $\lambda A e^{-Bx} = v(v^{-1}(\lambda A e^{-Bx}) \text{ and using (3) we prove this inequality.}$

Let us now study two prominent examples of specific utility functions, where the payoff function of the optimal structured product can be given explicitly.

Power utility

Let $u(x) := \frac{1}{\alpha}x^{\alpha}$ with $\alpha < 1$, $\alpha \neq 0$. Then $v(x) := u'(x) = x^{\alpha-1}$ and $v^{-1}(z) = z^{\frac{1}{\alpha-1}}$. Therefore, recalling (2), the optimal structured product is given by

$$y(x) = (\lambda \ell(x))^{-\frac{1}{1-\alpha}}.$$

We can compute λ explicitly, if we use the constraint (2):

$$\int (\lambda \ell(x))^{-\frac{1}{1-\alpha}} \ell(x) p(x) \, dx = R,$$

which can be resolved to

$$\lambda = \left(\frac{R}{\int \ell(x)^{\frac{\alpha}{\alpha-1}} p(x) \, dx}\right)^{\alpha-1}.$$

All together we obtain:

$$y(x) = \frac{C}{\ell(x)^{\frac{1}{1-\alpha}}}, \text{ where } C := \frac{R}{\int \ell(x)^{\frac{\alpha}{\alpha-1}} p(x) \, dx}$$

We see that y is a decreasing function of ℓ . In the case of the CAPM we obtain

$$y(x) = \frac{C}{(a-bx)^{\frac{1}{1-\alpha}}}.$$

Thus the structured product is an increasing and convex function of the underlying.

Exponential utility

Let $u(x) := -\frac{1}{\alpha}e^{-\alpha x}$. Then $v(x) := u'(x) = e^{-\alpha x}$ and $v^{-1}(z) = -\frac{1}{\alpha}\ln z$. Thus,

$$y(x) = -\frac{1}{\alpha} \ln(\lambda \ell(x)).$$

Again, we can compute λ explicitly, if we use 2, i.e.

$$\int y(x)\ell(x)p(x)\,dx = R.$$

The left hand side can be computed as follows:

$$\int y(x)\ell(x)p(x) dx = -\frac{1}{\alpha} \int \ln(\lambda\ell(x))p(x)\ell(x) dx$$
$$= -\frac{1}{\alpha} \int (\ln(\lambda) + \ln(\ell(x)))p(x)\ell(x) dx$$
$$= -\frac{1}{\alpha} \ln(\lambda) - \frac{1}{\alpha} \int \ln(\ell(x))p(x)\ell(x) dx.$$

Thus we obtain

 $\lambda = e^{-\alpha R - \int p(x)\ell(x)\ln(\ell(x)\,dx}.$

The optimal structured product is therefore given by

$$y(x) = R - \frac{1}{\alpha} \left(\ln \ell(x) - \int p(x)\ell(x) \ln(\ell(x)) \, dx \right).$$

And in the case of the CAPM we obtain

$$y(x) = R - \frac{1}{\alpha} \left(\ln(a - bx) - C \right),$$

where $C := \int p(x)(a-bx) \ln(a-bx) dx$. Again, y is an increasing and strictly convex function of x.

2.3 Optimal SPs in the general case

The assumption of strict concavity for the utility function is classical, but has been disputed a lot in recent years. Implied utility functions can be computed from stock market data and often show non-concave regions, compare for example [17] and [9]. Moreover, the most popular descriptive theory for decisions under risk, cumulative prospect theory, predicts non-concavity of u in losses [29]. Other descriptive theories also assume risk-seeking behavior at least for small losses.

Given the empirical and experimental evidence, it seems therefore more likely that u depends on a reference point (e.g. current wealth level) and that it has some strictly convex region for small losses. What would be an optimal SP for an investor described by such a model?

We first show that in order to find an optimal SP, it is sufficient to consider the *concavification* of the value function u, i.e. the smallest function larger or equal u which is concave⁸:

Lemma 1. Assume that u is concave for large returns and assume that the returns are bounded from below by zero⁹. Let u_c be the concavification of u, then there is an optimal SP for u_c which is also optimal for u.

Proof. Let y_0 be a point where $u_c(y_0) > u(y_0)$. We prove for any market return x_0 that an optimal SP for u does not have to yield the value y_0 , i.e. $y_0 \notin \operatorname{supp} y$. Suppose the opposite, i.e. $y(x_0) = y_0$ for some x_0 . We can find two points y_1, y_2 with $y_1 < y_0 < y_2$ such that $\lambda y_1 + (1 - \lambda)y_2 = y_0$ with $\lambda \in (0, 1), u(y_1) = u_c(y_1), u(y_2) = u_c(y_2)$ and $u_c(y_0) = \lambda u(y_1) + (1 - \lambda)u(y_2)$. Now we can construct another SP \tilde{y} such that whenever the market return is x_0, \tilde{y} gives a return of y_1 with probability λ and a return of y_2 with probability $1 - \lambda$. Let us do the same construction for all values y_0 where $u_c(y_0) > u(y_0)$. The new product obviously has a utility which is at least as big as before, since $\lambda u(y_1) + (1 - \lambda)u(y_2) = u_c(y_0) > u(y_0)$. Moreover, it satisfies by construction the pricing constraint. \Box

For the concavified utility we can now follow the same computations as before. The only difficulty is that the inverse of the derivative is still not everywhere defined, since it can be constant. This, however, simply corresponds to a jump in y, as can be seen, for instance, by an approximation argument.

This implies particularly that we typically do not have convexity of the optimal structured product in the case of non-concave utility functions. There

⁸A similar result has been obtained by Prigent [22]

⁹This is always true in assumptions since there is limited liability for structured products, i.e. the most one can lose is the initial investment.

is, however, one general property that holds in both cases which has been proved (in a more general setting) in [23]:

Theorem 3. If the decision model is given by an arbitrary utility function (possibly with reference point set by the initial wealth level and not necessarily concave), any optimal structured product in a CAPM market or a market with Black-Scholes pricing is a monotonic function (or correspondence) of the market return.

In other words: the higher the return of the market portfolio at maturity, the higher the return of the optimal structured product.

The intuition for this result is that whenever possible we would like to put large returns on "cheap" states.¹⁰ Since the state price density is decreasing in market returns, both in the case of CAPM and the case of Black-Scholes, the cheapest states are the ones with the largest market returns. This forces us naturally to assign large returns to states with large market return and consequently small returns to states with small market return.

2.4 Numerical results: what drives what?

We know now how optimal structured products can be analytically computed and what properties they satisfy. This framework should enable us to design optimal structured products for investors with a broad range of risk preferences. However, the numerical computation is not as easy as the previous examples suggested: in principle, we just need to evaluate equation (2), however, it is often difficult to compute the Lagrange parameter λ as explicitly as we have done in the case of a power and of an exponential utility function. Thus, we use for our numerical computation an iteration method, i.e. we evaluate (2) for a fixed λ , then compute the error of the constraint (2) and correct the λ . We iterate this until the error is sufficiently small.

Let us now give a couple of examples to see how optimal structured products look like.

Power utility

Let us consider the power utility function $u(x) = x^{\alpha}/\alpha$ (i.e. a typical function with constant relative risk aversion $-x \cdot u''(x)/u'(x) = 1 - \alpha$). The optimal structured products for expected utility investors with this utility function on a CAPM market (here and in the following examples always R = 0.05, $\sigma = 0.2$ and $\mu = 1.09$) is shown in Fig. 3. We see that the payoff function

 $^{^{10}}$ This intuition can be used to prove a special case of this theorem, which has been found by Dybvig [10].



Figure 3: Optimal structured product for CRRA-investors with $\alpha = -4, -2.5, -1, 0.5$.

of the product is strictly convex (which we know already from Theorem 1). It features a "smooth" capital protection and an increasing participation in gains.

Exponential utility

Let us consider the exponential utility function $u(x) = -e^{-\alpha x}/\alpha$ (i.e. a typical function with constant absolute risk aversion $-u''(x)/u'(x) = \alpha$). The optimal structured product for an expected utility investor with this utility function (for $\alpha = 0.5$) on a CAPM market is shown in Fig. 4. We see that the payoff function of the product is only very slightly convex.

Quadratic utility with aspiration level

Let us now consider a non-concave utility function. Non-concave utility functions are a key ingredient of some behavioral decision models, like prospect theory. They can also occur, however, for very rational reasons: an example would be an investor who plans to buy a house in one year and has saved just about enough money for the installment. (We denote his current wealth level by x_0 .) His utility function, when considering a one-year investment, will now be necessarily have a jump (maybe slightly smoothed by the uncer-



Figure 4: Optimal structured product for an CARA-investor with $\alpha = 0.5$.

tainty about the house prices) that will make it locally non-concave.¹¹ Let us define the utility function as v(x) = a(x) + u(x) with $a(x) = h(x)u_h$, where u_h is the extra utility gained by the house and h(x) is the probability that he is able to afford the house which we set for simplicity as one for $x \ge x_0$ and zero for $x < x_0$, thus inducing a jump in the utility function at x_0 . In reality the probability might be more like a logistic function rather than a precise jump function, thus the overall utility v would look more like a concave–convex–concave function. See Fig. 5 for illustrations.

To keep things simple we choose as u the function $u(x) = x - \alpha x^2$ with $\alpha = 0.2$. (Larger values of α lead to an saturation problem.) Moreover we set $x_0 = 1$ and $u_h = 0.02$. We need to compute the concave hull of

$$v(x) = a(x) + u(x) = \begin{cases} x - \alpha x^2, & \text{for } x < 1, \\ 0.02 + x - \alpha x^2, & \text{for } x \ge 1 \end{cases}$$

To this end, we compute the derivative of u to find the tangential line on u which crosses the point (1, v(1)). A straightforward computation using (2) leads to an explicit solution, depending on the Lagrange parameter λ which is then computed numerically by an iteration scheme as outlined above.

The resulting optimal structured product is piecewise affine (as was to be

¹¹This is similar to the aspiration level in the SP/A model, see [26, Chapter 25.5] for an overview and further references.



Figure 5: The utility function v of an investor with an aspiration level: his utility increases above a certain threshold, thus making it non-concave with a jump (left). If the precise location of the threshold is uncertain, the effective utility function becomes again continuous, but is still convex around that point (right).



Figure 6: Optimal structured products for investors with quadratic utility ($\alpha = 0.2$) plus an aspiration for $u_h = 0.2$ at position $x_0 = 0.9, 1, 1.05, 1.1$.

2.4 Numerical results: what drives what?

expected when using a quadratic u in CAPM¹²) and corresponds to a limited capital protection as it can be found in many structured products (compare Fig. 6).

The model, even though certainly simplistic, can in fact be confirmed by empirical evidence: the classical investment goal for private investors is real estate. Whereas in the US the percentage of realty owners in the population is very high, this is not the case in most continental European countries. moreover there houses are usually owned for longer periods of time, thus, buying realty tends to be a very special event. If our previous considerations are correct, then we would therefore expect that investors who plan to buy realty in the next future would be more interested in buying capital protected SPs. This has been tested in a survey by AZEK [12], where N = 106 test subjects have been asked to choose between six structured products on the MSCI World, out of which one provided a full capital protection. The test subjects were mainly bankers and other professionals working in the financial industry, thus their competence in understanding the survey question can be assumed. The subjects were also asked whether they plan to buy in the next years a house or an apartment and whether they already own realty. A simple lottery question was used as proxy for their loss aversion.

According to our model there should be a significant positive relation between the investment into a capital protected product and the plan to buy realty. This correlation could, however, also be triggered by a general affinity to conservative investments, which is just reflected by the plan to buy realty. To test this, we performed a logit regression where we added two independent variables: the loss aversion as estimated from the lottery question and a dummy variable for the owners of realty. It turned out that all of these factors were significant for their decision (see Table 2.4). The effect was stable when removing the dummy variables for realty owners and the loss aversion and was even more pronounced when only considering subjects which did not own realty yet (N = 85).

Prospect theory

We can also use prospect theory as underlying preferences, i.e. a utility function which is convex in losses and concave in gains. As functional form we use here the specification from [33] that can be seen as a second order

¹²If we use Black-Scholes, we get piecewise affine solutions when using piecewise exponential utility functions, as can be shown be a similar computation.

All subjects $(N = 106)$:						
	Model 1		Model 2		Model 3	
	Coeff.	$\text{Prob} > \chi^2$	Coeff.	$\text{Prob} > \chi^2$	Coeff.	$\text{Prob} > \chi^2$
Plan to buy	1.994	0.001***	0.749	0.047^{**}	1.101	0.059^{*}
Owning realty	1.919	0.032^{**}	1.138	0.066^{*}		
Loss aversion	0.819	0.035^{**}				
		Only non-ov	where $(\Lambda$	V = 85):		
	М	odel 1	М	odel 2		
	Coeff.	$\text{Prob} > \chi^2$	Coeff.	$\text{Prob} > \chi^2$		
Plan to buy	1.845	0.002^{***}	1.907	0.001^{***}		
Loss aversion	1.094	0.001^{***}				

Table 1: Factors influencing the decision to invest into a capital protected structured product. *, **, ** =significant on the 10%, 5%, 1% level.

approximation of any prospect theory value function:¹³

$$u(x) := \begin{cases} -\lambda(x - \beta x^2), & \text{for } x < 0, \\ x - \alpha x^2, & \text{for } x \ge 0. \end{cases}$$
(5)

We can think here of an investor who is (like most investors) afraid of losses, but does not distinguish that much between small and large losses. This leads to risk-seeking behavior in losses. Depending on the amount of loss-aversion λ a smaller or larger amount of capital protection becomes optimal. Figure 7 shows the optimal solution in this case for various values of loss-aversion.

2.5 Utility gain by SPs: much ado about nothing?

We have seen so far that optimal structured products do indeed deliver a payoff structure that is different to classical portfolios including only the market portfolio and a risk-free asset. The effort of computing such an optimal structured product and hedging it can in praxis not be neglected, therefore the natural question arises whether it is worth it: how big is the potential utility improvement?

To answer this question, we compute the expected utility of an optimal structured product and the expected utility of the optimal mix between the market portfolio and the risk-free asset. We translate both values into certainty

¹³This form is also computationally simpler than the standard specification by Tversky and Kahneman [29] and incorporates mean-variance preferences as a the special case when $\alpha = \beta$ and $\lambda = 1$.



Figure 7: Optimal structured products for investors with a prospect theory utility as in (5) with $\alpha = \beta = 0.2$ and $\lambda = 1, 1.5, 2, 2.5$.

equivalent interest rates, i.e. the (hypothetical) risk-free asset that would have the same expected utility as the product respectively the portfolio. We then consider the difference between both certainty rates and compare it with the gain in certainty rates that can be achieved by a classical portfolio over a risk-free investment. This analysis shows how good the "second order approximation" (the classical two-fund portfolio) is, comparing to the "higher order approximation" (the optimal structured product). Thus we want to answer a couple of questions of practical relevance: is the first (classical) step of improvement big, and the second one (caused by the structured product) negligible? Or are they equally important? Under what circumstances is the additional potential improvement of structured products particularly large? To keep the analysis simple, we restrict ourselves to the two examples of the previous section: first, we consider a CRRA-utility function of the form $u(x) = x^{\alpha}/\alpha$. Later we will study the case of a quadratic utility function with aspiration level.

The optimal structured product for CRRA-utility functions has been already computed (see Section 2.2). Thus we can directly compute the expected utility of y, where we assume a normal distribution with mean $\mu = 1.08$ and standard deviation $\sigma = 0.19$ for the return of the market portfolio. Reporting only this value would be idle: we need to compare it with other utilities. Therefore we compute the improvement, as expressed in terms of certainty equivalent interest rate, over the optimal "classical" portfolio, i.e. the optimal combination of the market portfolio and a risk-free investment. To compute the optimal classical portfolio, we compute the utility of all portfolios with a proportion of θ invested in the market portfolio and $(1 - \theta)$ invested into risk-free assets, where $\theta \in \{0, 0.01, \ldots, 0.99, 1\}$. Then we choose the θ yielding the largest expected utility.

The results of this computation are summarized in Table 2.5.

Investment	Certainty equivalent
Fixed interest (4%)	4.00%
Market portfolio (optimal classical)	4.58%
Optimal structured product	4.64%

Table 2: Improvement of a structured product for an investor with a classical CRRA utility with $\alpha = -0.2$ as compared to the improvement by classical portfolios.

We see that the optimal structured product gives an improvement, but it is not as big as the "first step", i.e. the improvement induced by the classical mean-variance portfolio theory. The improvement over the classical portfolio is only 6 basis points.

This is not due to a specific choice of α . Very much to the contrary, Fig. 8 shows that the improvement is in fact small for all natural choices of α .

Investment	Certainty equivalent
Fixed interest (4%)	4.00%
Optimal classical portfolio	4.24%
(53% market portfolio, $47%$ fixed interest)	
Optimal structured product	4.24%

Table 3: Improvement of a structured product for an investor with a classical CARA utility with $\alpha = 0.01$ as compared to the improvement by classical portfolios.

The second example, an exponential utility function, can be computed in the same way. Here the improvement is minute (only 0.0004%) and in the precision of the data in Table 2.5 not even visible. This does not come as a big surprise, since the optimal structured product is close to a linear investment (compare Fig. 4). The result is very similar if we vary the absolute risk aversion.



Figure 8: Utility for CRRA (as measured by the certainty equivalent interest rate) of stock, risk-free asset (5% return), optimal two-fund portfolio and optimal structured product. The utility gain by the structured product is small.

So far it looks like the improvements of structured products are somewhere between tiny and small, but noticeable. Structured products – much ado about nothing? Are there no situations in which we can generate a decisive improvement of a portfolio with their help?

Let us consider the third example, the quadratic utility with aspiration level. Here, finally, the improvement due to structured products is considerable as we see in Table 2.5. In fact, the improvement is as big (or bigger) than the "first step improvement" done by the classical mean-variance theory! The improvement is similar for various "aspiration levels" as Fig. 9 illustrates.

Investment	Certainty equivalent
Fixed interest (4%)	4.00%
Market portfolio	3.74%
Optimal classical	4.06%
(8% market portfolio, 92% fixed interest)	
Optimal structured product	4.30%

Table 4: Improvement of a structured product for an investor with an aspiration level as compared to the improvement by classical portfolios.



Figure 9: Utility with aspiration level (as measured by the certainty equivalent interest rate) of stock, risk-free asset (5% return), optimal two-fund portfolio and optimal structured product. We have varied the position of the jump in the utility, i.e. the wealth the investor needs to reach his investment goal. The utility gain by the structured product corresponds here up to 1% interest rate.

Let us finally consider a prospect theory investor (without probability weighting) with a utility function as in (5) with $\alpha = \beta = 0.2$. We consider his utility improvement for various levels of loss aversion λ , see Fig. 10.

To summarize our results it seems that whereas in classical strictly concave settings, the additional amount of risk control due to structured products is not large and, depending on the utility function, sometimes even quite small. As soon as we broaden our horizon and look at situations with partially nonconcave utility functions, structured products become much more interesting. In such situations, they can easily improve the portfolio by an amount that is larger than the improvement of the first step from a fixed interest rate to a classical optimal portfolio á la CAPM. However, the banks typically charge at least 1% on SPs hence the utility improvements we found are not worth these costs.



Figure 10: Prospect utility (as measured by the certainty equivalent interest rate) of stock, risk-free asset (5% return), optimal two-fund portfolio and optimal structured product. We have varied the loss aversion λ of the investor. The utility gain by the structured product becomes substantial for low loss aversion and is still in the order of 1.5% for interest rate for a normal loss aversion of around 2.

3 After all, why do people buy structured products?

So far our aim was to find optimal structured products for given utility functions and characterize the shape of the payoff diagram and the size of the utility gain. In this section we want to find for given structured products (that we observe, e.g., on the market) a utility function and other aspects for which these products are optimal.

As seen above an investor with expected utility theory that is strictly increasing, strictly concave and prudent, i.e. a classical rational investor would only invest into SPs with a strictly convex payoff function. This is in striking contrast to the variety of structured products on the market as we have seen them in Fig. 1 and Fig. 2.

In this section we want to investigate other potential reasons that can make structured products attractive for investors, thereby relaxing our assumptions on a classical rational investor step by step.

3.1 Background risk

Let us relax our initial assumptions that an investor only invests into a structured product (or considers the investment in SPs separately from other investments) and allow the investor instead to hold additionally to the SP a portfolio of classical assets – a combination of the market portfolio and risk-free assets. In order to conclude that also in this setting optimal structured products are convex, suppose the contrary, i.e. that the return of the SP is described by a function y which is non-convex. Then the overall portfolio can be described by the function

$$\tilde{y}(x) = \lambda_1 R + \lambda_2 x + (1 - \lambda_1 - \lambda_2) y(x),$$

where $\lambda_1 > 0$ is the proportion of total wealth invested in risk-free assets and $\lambda_2 > 0$ is the proportion of wealth invested in the market portfolio such that $1 - \lambda_1 - \lambda_2 > 0$. According to Theorem 1 the function \tilde{y} is strictly convex, but $\tilde{y}''(x) = y''(x)$, thus we have a contradiction to the assumption that y is non-convex.

We conclude that a non-convex SP for a classical rational investor would only be useful if he already has assets with a strictly convex payoff function in his portfolio. Mixing his assets in this way seems unrealistic, but ignoring this, if we consider such assets as SPs and study the sum of all SPs, the statements about optimal SPs are still true, but apply to the sum of the SPs in the investor's portfolio.

3.2 Investors with PT-type utility function

There is ample experimental and empirical evidence in favor of non-classical decision theories like prospect theory. In the next step, we will therefore implement one of the key ingredients of prospect theory, namely the convexconcave structure of the utility function with respect to a reference point that is in itself not fixed, but can, for instance, be the initial value of an asset.

We have seen already that in this case loss-aversion can induce a non-convex payoff function of an optimal structured product with a "plateau" at zero return. This mimics a frequent feature of structured products, namely limited capital protection that is valid only up to a certain amount of losses.

More popular structures, however, like the highly popular barrier reverse convertibles can still not be optimal, since we have the following corollary from the co-monotonicity results in [23]:

Corollary 2. If the decision model is given by an arbitrary utility function (possibly with reference point set by the initial wealth level), any optimal

3.3 Probability weighting

structured product in a CAPM market (or any market where the likelihood ratio is decreasing as a function of the return of the market portfolio) is a monotonically increasing function (or correspondence) of the market return.

In other words: the higher the return of the market portfolio at maturity, the higher the return of the structured product.

This is obviously violated for all types of barrier products or, more generally, all structured products whose return at maturity is not solely defined by the return of the underlying market portfolio at maturity. Thus the modified utility function of prospect theory can only explain the popularity of some of the structured products offered today. The most popular types cannot be understood within this model.

3.3 Probability weighting

Experimental studies have demonstrated that subjects systematically overweight small probabilities, i.e. small probability events (e.g. large losses when deciding about buying insurance or large gains when deciding about buying a lottery ticket) tend to have more impact on the decision than they ought to have [18]. This effect should not be mixed up with probability misestimation: probability weighting even affects decisions where the probabilities are known and do not have to be estimated. In this section we add probability weighting and observe its effects on the qualitative features of optimal structured products.

First, we need to distinguish two models of probability weighting:

- 1. We consider the return distribution of the structured product and apply a probability weighting to it.
- 2. We apply probability weighting to the return distribution of the underlying.

Both approaches are a priori reasonable, but lead to very different results. In particular, in the first case, the monotonicity result of Theorem 3 still holds, thus the payoff function is still monotonic. In the second case, however, this is no longer the case, since the utility becomes in a certain sense state dependent. With this form of probability weighting we can therefore explain, for instance, the attractiveness of constructions that give high payoffs for extreme events (which are overweighted by the probability weighting), in particular straddles.

Nevertheless, the payoff is still (essentially) a function of the underlying:

Theorem 4. Assume that the return of the market portfolio is an absolutely continuous measure with a smooth distribution function p which is nowhere equal to zero. For an investor with a smooth utility function (possibly with a reference point depending on the initial wealth) and a probability weighting of the underlying with a smooth probability weighting function w, an optimal structured product has a piecewise smooth payoff function¹⁴ of the underlying market portfolio.

Proof. Let p_* denote the return distribution of the market portfolio after probability weighting has been applied. More precisely, in cumulative prospect theory we define¹⁵

$$p_*(x) := \frac{d}{dx} \left(w \left(\int_{-\infty}^x p(t) \, dt \right) \right).$$

Similarly, in prospect theory (see [18, 19, 25]), we define for $\gamma \in (0, 1]$:

$$p_*(x) := \frac{p(x)^{\gamma}}{\int_{\mathbb{R}} p(x)^{\gamma} \, dx}.$$

Now in both cases it is easy to see that p_* is smooth and nowhere zero. We define $w_*(x) := p_*(x)/p(x)$. Then the optimal structured product maximizes

$$U(p) := \int_{\mathbb{R}} u(y(x)) p_*(x) \, dx,$$

subject to

$$\int_{\mathbb{R}} y(x) \frac{\ell(x)}{w_*(x)} p_*(x) \, dx = R.$$
(6)

From the co-monotonicity result [23], we know that the maximizer y of this problem is a monotonic function (remember that p is absolutely continuous) of $\ell(x)/w_*(x)$. Since w_* is smooth, we can differentiate this expression and arrive at

$$\frac{d}{dx}\left(\frac{\ell(x)}{w_*(x)}\right) = \frac{\ell'(x)w_*(x) - \ell(x)w_*'(x)}{w_*(x)^2},$$

which is a smooth function, thus y is piecewise smooth.

This result implies in particular that y is still a *function* of the market return. This has a strong consequence: some of the most popular classes of products, namely all products with *path-dependent* payoff (in particular barrier reverse convertibles and bonus certificates), still cannot be explained by this model!

 $^{^{14}\}mathrm{This}$ means that the payoff function can have finitely many jumps, but is smooth everywhere else.

¹⁵We could also use a decumulative function in gains as in the original formulation in [29], but both is mathematically equivalent if we choose w differently in gains and losses.

3.4 Betting against the market

We have seen from the results in the previous sections: if we want to understand why investors buy structured products like barrier reverse convertibles or basket products, we need to take into account other factors than just risk preferences.

One such factor is disagreement with the market beliefs: so far we have always assumed that investors don't know better than the market and thus the physical return distribution p is also used in their estimate of the utility. If both are different, we label this as "betting against the market" or "speculation". Believing that the market will behave differently than the probability distribution p forecasts might be wise in some circumstances, but it is probably much more frequently a sign of overconfidence – an all too common characteristics of private investors.

First, we notice that the monotonicity result (Theorem 3) does not hold anymore, since it relied on the homogeneity of the beliefs. In fact, we have to change our model. Fortunately, we can use essentially the same idea as with probability weighting: let us denote the estimated probability of the investor by \tilde{p} , then his optimization problem becomes

Maximize
$$U(y) := \int_{\mathbb{R}} u(y(x))\tilde{p}(x) dx$$
,

in $L^1(\mathbb{R}, \ell p)$ subject to

$$\int_{\mathbb{R}} y(x)\ell(x)p(x)\,dx = R.$$
(7)

Defining $w(x) := p(x)/\tilde{p}(x)$ and $\tilde{\ell}(x) := w(x)\ell(x)$ we can transform this problem to

Maximize
$$U(y) := \int_{\mathbb{R}} u(y(x))\tilde{p}(x) \, dx$$
,

subject to

$$\int_{\mathbb{R}} y(x)\tilde{\ell}(x)\tilde{p}(x)\,dx = R.$$
(8)

This new problem can now be solved in the same way as above. As with probability weighting, we do not necessarily have a monotonic payoff function, but the fact that the optimal SP can be described by a payoff function still holds: **Corollary 3.** Assume that the return of the market portfolio is an absolutely continuous measure with a smooth distribution function p which is nowhere equal to zero. Let \tilde{p} be a smooth return distribution estimated by the investor. Assume that the investor has a smooth utility function then an optimal structured product is a piecewise smooth function of the underlying market portfolio.

Proof. The proof follows the same idea as the proof of Theorem 4. Moreover, we can give a condition on w under which the optimal y is monotonic if we assume that ℓ is positive and decreasing (as, e.g., in the Black-Scholes model or in CAPM for not exceedingly large returns): using the co-monotonicity we know that y is monotonic whenever $\tilde{\ell}$ is a decreasing function. This is the case if $\tilde{\ell}'(x) = \ell'(x)w(x) + \ell(x)w'(x) < 0$. Since w(x) > 0, $\ell'(x) < 0$ and $\ell(x) > 0$, this holds particularly when w'(x) < 0, i.e.

$$p(x)'\tilde{p}(x) + p(x)\tilde{p'}(x) < 0.$$

$$\tag{9}$$

Example 1 (Optimism and pessimism). Let p and \tilde{p} be normal distributions. If the investor is optimistic, i.e. var $p = \operatorname{var} \tilde{p}$, but $\mathbb{E}(p) < \mathbb{E}(\tilde{p})$, then condition (9) is satisfied, thus the optimal y is an increasing function of the market index. If $\mathbb{E}(p) > \mathbb{E}(\tilde{p})$, the condition is violated, thus y may be non-monotonic.

Example 2 (Over- and underconfidence). Let p and \tilde{p} be normal distributions. If the investor is overconfident in the sense of too narrow probability estimates, i.e. $\operatorname{var} p > \operatorname{var} \tilde{p}$, but $\mathbb{E}(p) = \mathbb{E}(\tilde{p})$, then condition (9) is satisfied for all $x > \mathbb{E}(p)$, thus the optimal y is an increasing function of the market index for $x > \mathbb{E}(p)$. If $\operatorname{var}(p) < \operatorname{var}(\tilde{p})$, this is the case for $x < \mathbb{E}(p)$.

Figure 11 shows examples of optimal SPs for CRRA investors ($\alpha = -2$) with different beliefs.

3.5 Probability mis-estimation

Betting against the market can explain the popularity of some investments (like yield enhancing (call option style) or contrarian (put option style) products), but it seems less likely that it can explain the popularity of complicated constructions like barrier reverse convertibles. Here a better explanation is probability mis-estimation. The difference to betting is that it is not caused by a difference in beliefs, but rather a difficulty to translate (potentially accurate) estimates on the market into correct estimates on probabilities for certain events that are important for the payoff of a structured product.



Figure 11: Optimal payoff diagrams for investors with pessimistic to optimistic estimations. $\mathbb{E}(\tilde{p}) = 1.00, 1.05, 1.09, 1.13$ and $\mathbb{E}(p) = 1.09$.

As an example we consider products with a barrier (e.g. barrier reverse convertibles). These are products that have a capital protection that vanishes when the price of the underlying *at some point before maturity* falls below a certain threshold ("barrier"), compare Fig. 13a. These products are very popular, as we have seen, although their payoff at maturity is not a function of the final price of the underlying and thus they are not optimal for any decision model (see above), even though a prospect theory or aspiration level like utility comes close to this payoff profile.

Probability mis-estimation can explain this popularity: it has been experimentally demonstrated in [23] that test subjects underestimate the probability that the barrier is hit at *some point in time* before maturity with respect to the probability that the prize is below the barrier level *at maturity*. This makes barrier reverse convertibles seemingly more attractive and can thus explain why behavioral preferences alone (without modeling the misestimation of probabilities) cannot explain their popularity.

This is not the only class of popular structured products that conflicts with the rational and behavioral frameworks we have developed. Another important example where probability misestimation seems to play an important role are basket products, e.g. products which return in certain situations the value of the worst performing stock of a predefined basket of assets. Experimental work on this class of products is ongoing [24].

3.6 A classification of structured products

Our results can be used to classify SPs according to their attractiveness for rational and irrational investors. In the "family picture" of structured products we mark certain properties that our analysis found important for the theoretical attractiveness of structured products, see Fig. 12 and Fig. 13). Here we assumed (in favor of the investors' rationality) that all SPs have the market index as underlying¹⁶.

In this way, we can classify all SPs into several categories, according to the degree of deviation from rational preferences that we need to explain their attractiveness to investors. The overall conclusion is that at least 17 30.5% of the issued SPs in Germany and 48.6% in Switzerland have jumps in their payoff diagrams (pointing to non-concave utility functions of investors), 3.1% (resp. 25.8%) have non-monotonic payoff functions¹⁸ (pointing to heterogeneous beliefs), 36.2% (resp. 48.6%) cannot be described by a function of the underlying (suggesting systematic probability misestimation as investment motif), finally, 76.6% (resp. 61.6%) have non-convex payoffs (pointing to increasing absolute risk aversion), see Table 6 and 7 (appendix) for details. We could sum up these examples by saying that the most popular classes of structured products combine in a clever way prospect theory-like preferences (in particular loss-aversion and risk-seeking behavior in losses) and probability mis-estimation induced by a complicated payoff structure that leads to a systematic underestimation of the probability for unfavorable outcomes. Hence we have argued that SPs reflect the importance of non-traditional utility functions and of behavioral biases.

4 What structured products would investors choose if they understood them?

Taking real sales or emission data of structured products as foundation for theories on investor preferences is in so far limited as it does neither take into account the marketing efforts of banks, that may substantially vary for dif-

 $^{^{16}}$ In fact, this is a very optimistic assumption, since most have single assets or even worst-of baskets as underlying which makes is even clearer that betting and misestimation play pivotal roles in investment decisions for structured products: for the German market we know, e.g., that as of November 2007 only 25.0% of the structured products used a market index as underlying.

¹⁷Products with unclear payoff profile could increase these numbers.

¹⁸Contrary to the previous sections, we consider a payoff monotonic if the payoff as function of the underlying is increasing in *each scenario*, thus also barrier products can be considered as monotonic in this sense. Otherwise the numbers would be much higher.



co-monotone; non-convex; can be explained by increasing ARA



(b) Bonus-/partially protected products

non-co-monotone; can only be explained by combination of non-concave utility and probability misestimation



(c) K.O. products with stop loss

can be explained by speculation





(d) K.O. products without stop loss can be explained by specu-

lation

co-monotone; non-convex; can be explained by increasing ARA

(f) Exotic leverage

co-monotone; non-convex; can be explained by increasing ARA

Figure 12: The list of the most popular structured products on the German market and an explanation of their attractiveness, based on our results.

ferent product types, nor the availability of structured products. Laboratory experiments on the other hand suffer from sample selection (typically they are conducted with undergraduate students and not with real investors) and from a lack of real incentive. We had the unique chance to conduct a field study and to collect data in a rather unusual way: during the 175th anniversary event of our university a week-long exhibition at a central location in downtown Zurich took place where we presented a multi-touch table for the design of structured products.¹⁹ For a screenshot of the structured product

¹⁹The hardware and the user interface had been designed for us by the Swiss Design Institute for Finance and Banking. The core programming was done by the institute for computer science at the University of Zurich.





(a) Barrier reverse convertibles

non-co-monotone; can only be explained by increasing ARA plus probability misestimation (b) Tracker certificates

co-monotone; convex; optimal for CARA (in Black-Scholes model)



(c) Bonus certificates

non-co-monotone; can only be explained by combination of non-concave utility and probability misestimation



Figure 13: The same illustration for the Swiss market.

editor see Fig. 18 in the appendix. The visitors who tried our multi-touch table were obviously self-motivated and typically interested in financial investments. N = 322 persons stated their gender out of which 69.9% were male and 30.1% female. All age groups were represented (average age was 35 years, standard deviation 17 years).

The program introduced the concept of payoff diagrams and structured products, then went on to let the visitor design a structured product by interactively editing its payoff diagram. The underlying was fixed as the SMI with one year maturity, pricing according to Black-Scholes. Backtesting and re-editing was possible, see the appendix for details, thus we can be confident that the participants understood well what structured products are and how they work. The resulting payoff curves were recorded, together with the visitors' answers to three lottery questions (as proxy for their risk attitudes), reasons why they might want to invest into structured products and some demographic data (gender, age group and education).

The total data comprised N = 684 data sets, out of which N = 233 were complete (including answers to the demographical questions and the lottery tasks). Due to the uncontrolled situation at the exhibition it is not surprising that there were a few "playful" designs, so after data cleaning (taking out SPs with excessive fluctuations, i.e. volatility above 50) N = 219 out of the 233 complete data sets remained.

What are the characteristics of the designed products? First, we checked for properties that optimal SPs should satisfy according to the theory. Since we assumed a payoff function on the SMI we avoided a large class of non-optimal products. Nevertheless 47.7% of the self-designed SPs were non-monotonic (pointing to betting as motif) and 9.2% (4.7% of the monotonic products) showed a clear jump (pointing to non-concave utility functions). We also asked participants to state how much three potential reasons (diversification, portfolio insurance, speculation) would contribute to their decision to buy structured products where we used the same questions as in [11]. The most popular motif in our sample was portfolio insurance with an average rate of 4.0 on a scale from 1 (not at all important) to 5 (very important). However, 45.2% of the participants gave 4 or more points to speculation, underlining the importance of betting for explaining attractiveness of structured products.

Product type	Chosen by
Capital protection without cap	49.0%
Capital protection with cap	18.9%
Straddles	6.8%
Tracker certificate (bear)	5.8%
Outperformance certificate with cap	3.9%
Outperformance certificate without cap	2.9%
Tracker certificate (bull)	1.9%
Others	10.8%

Table 5: Choices of structured product categories by the participants of our field study. Complex products (e.g. products with barriers) could not be constructed with our tool and are consequently missing in this list.

Manual matching the SPs to classical categories resulted in a distribution of the most popular types as shown in Table 4, giving yet another family picture of structured products, that by construction excludes all barrier products and similarly complex structures.

In the next step, we averaged over all monotonic payoff functions to get a simple proxy for the favorite structured product of the "representative agent" (see Fig. 14) which looks similar to a capped capital protected product and has features of a "covered call" which had been found to be popular among investors buying options (compare [27] for an empirical study). The payoff pattern points to increasing absolute risk aversion in gains and decreasing absolute risk aversion in losses (compare Theorem 2).



Mean of Monotonous Structured Products

Figure 14: The average structured product as designed by the participants of our experiment (line) with 95% confidence intervals (circles). Only products with a monotonic payoff have been considered to take out obvious instances of speculative motifs.

It is possible to compute the corresponding utility function (or more precisely its concave envelope which we denote in the following for simplicity by u) for which this SP is optimal. To this end we need to take the inverse approach of the optimization procedure. We start from the derivation of (2) where we had $u'(y(x)) = \lambda \ell(x)$. Since the preferences of expected utility functions are invariant under affine transformation, we can set $\lambda = 1$, thus we obtain the following result:

Remark 1. Let y be the optimal structured product for an expected utility maximizer with market beliefs, then the concave envelope of his utility function can be computed as

$$u'(z) = \ell(y^{-1}(z)).$$

Assuming that $\ell(x)$ is strictly decreasing in x and positive (as in Black-Scholes), we can derive that a jump in y corresponds to a convex part of u (i.e. u' is constant). A capped y (i.e. y is constant above a certain threshold x_0) corresponds to u constant above $y(x_0)$.

In our experiment the pricing was computed with the Black-Schoels model, thus we assume $\ell(x) = Ae^{-Bx}$ with A, B > 0. It is now easy to compute u' (and hence u) from y. As an example let us consider the case y(x) = x. Then $u'(x) = Ae^{-Bx}$, i.e. $u(x) = -(A/B)e^{-Bx}$, i.e. a direct investment in the stock market is optimal for an investor with CARA utility.²⁰

We can do the same computation for the average SP from our experiment and obtain the concave envelope of the utility as illustrated in Fig. 15. In Fig. 16 we plot the corresponding Arrow-Pratt risk aversion -u''(x)/u'(x)(scaling is return⁻¹). It is decreasing for losses and increasing for gains – as expected by the convex-concave shape of the SP. Figure 17 shows the relative risk aversion $-x \cdot u''(x)/u'(x)$ where we assume that the complete wealth is invested into the structured product. It is decreasing in the loss domain and increasing in the gain domain with a minimum value around 2. Assuming that only half of the wealth are invested in the structured product and the rest is invested risk-free would correspond to a minimum relative risk aversion of around 4 (corresponding to $\alpha = -3$ if $u(x) = x^{\alpha}/\alpha$).

One might wonder whether the strict concavity of the utility that we computed does not imply that non-concave utility functions were not needed in our analysis. However, we remind that the concavity only holds for the average u where non-concavities of individuals average out.

²⁰It is interesting to compare this result with the case of CAPM: there the optimal SP was an affine function of the market index for an investor with quadratic utility, i.e. mean-variance preferences.



Figure 15: The expected utility function implied by the average structured product from our experiment where only products with monotonic payoff have been considered.



Figure 16: The absolute risk aversion of the average structured product from our experiment: decreasing in losses, increasing in gains.



Figure 17: The relative risk aversion of the average structured product from our experiment: decreasing in losses, increasing in gains (under the assumption that all wealth is invested into this product).

5 Conclusions

We have seen that structured products can arise as a solution to enhancing the performance of a portfolio. Depending on the (rational) risk attitudes of an investor it is usually good to use a product that leads to a strictly convex payoff structure for the risky part of the portfolio. This can be done by hedging against losses of different degrees. We estimate the size of the improvement when comparing to a classical Markowitz-style (mean-variance) investment and found that the improvement is typically smaller than the costs of structured products.

Most popular structured products, however, do not follow this rational guideline, but instead use behavioral factors, like loss-aversion or probability misestimation to be attractive in the eyes of potential investors. In particular we could show that the currently most popular products clearly cannot be explained even within the framework of prospect theory, but only when taking into account probability mis-estimation. Thus we come to the conclusion that by and large the market for structured products, which is a huge business for banks, offers a utility gain for investors which is most likely only an illusion. Instead of banning structured products completely (as it is currently discussed in some countries), we would suggest to improve the understanding of customers (as we have done, e.g., with our multi-touch table application). Also it would be wise to introduce independent rankings that enable specific types of investors to see whether structured products add value to their portfolio or not.

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A Screenshot of the SP editor

Figure 18: A screenshot of the structured product editor (a single-touch version can be accessed online at http://www.sdfb.ch/projects/2008/speditor/). The curve can be manipulated by dragging the little balls and is thereby auto-adjusted according to the Black-Scholes formula. Mean and standard deviation ("Mittelwert" and "Standardabweichung") of the underlying Swiss Market Index were displayed and explained. The test subjects could run simulations for various years (2000–2007) to backtest their product before settling for a final design.

B Number of issued structured products in Germany and Switzerland

Category	Issued
Increasing, convex function of underlying, no jumps	5
(risk optimization, non-increasing ARA)	
Guarantee certificates	1,857
Index certificate	$1,\!998$
Outperformance certificates	2,219
Subtotal	3.5%
Same but not convex (risk optimization, increasing AF	RA)
Reverse convertible	$6,\!155$
Discount certificates	66,511
Subtotal	42.3%
Not necessarily increasing (potentially speculation)	
Exotic leverage	5,319
KO products with stop loss	$24,\!161$
Subtotal	17.2%
Several scenarios, in each scenario increasing function of un	derlying
(probability misestimation)	
Basket certificates	1,569
Bonus-/partial protection cert.	$52,\!321$
Subtotal	31.4%
Not classifiable	
KO product w/o stop loss	8,281
Others	1,434
Subtotal	5.7%

Table 6: Approximate distribution of structured products issued in Germany according to normative and behavioral categories.

Category (according to SVSP classification [1].)	Issued
Increasing, convex function of underlying, no jumps	
(risk optimization, non-increasing ARA)	
Tracker Certificate	8513
Outperformance Certificate	375
Uncapped Capital Protection	2983
Capital Protection with Coupon	170
Subtotal	25.4%
Same but not convex (risk optimization, increasing ARA))
Airbag Certificate	58
Discount Certificate	3750
Reverse Convertible	1376
Capped Outperformance Certificate	123
Capped Capital Protection	303
Subtotal	11.8%
Not necessarily increasing (potentially speculation)	
Spread Warrant	61
Mini-Future	4335
Subtotal	9.3%
Several scenarios, in each scenario increasing function of under	rlying
(pointing to probability misestimation)	
Knock-Out Warrant	2224
Bonus Certificate	4666
Barrier Discount Certificate	407
Barrier Reverse Convertible	14428
Capped Bonus Certificate	494
Subtotal	46.9%
Partially decreasing function of underlying (speculation)	
Barrier Range Reverse Convertible	476
Twin-Win Certificate	342
Subtotal	1.7%
Not classifiable:	
Diverse Leverage, Diverse Participation, Express Certificate,	
Diverse Yield Enhancement, Diverse Capital Protection, Diverse	Product
Subtotal	4.8%

Table 7: Approximate distribution of structured products issued in Switzerland according to normative and behavioral categories.