Optimal Design of Rating Trigger Step-Up Bonds: Agency Conflicts Versus Asymmetric Information

April 2008

Abstract

In this paper, we analyze corporate bonds with a rating-triggered step-up provision in a continuous-time framework with bankruptcy costs and tax benefits. While without any further frictions, step-up bonds do not add firm value relative to straight debt, agency conflicts and asymmetric information are two possible explanations for the issuance of these instruments. We treat both motives (separately) in a unified framework to obtain conclusions about both the optimal design and the conditions for the use of step-up bonds. The closed-form solutions for the optimal contract design reveal that step-up bonds issued by firms that face a risk-shifting problem fundamentally differ from those in the case of asymmetric information. Furthermore, we show that firms with a high initial risk only use step-up bonds to overcome problems of asymmetric information but not to mitigate risk-shifting problems. A further difference between the two motives is that in the case of risk-shifting, step-up bonds are only used when the agency conflict is sufficiently severe, while for signalling reasons even a modest problem of asymmetric information supports the use of step-up bonds.

JEL classification: G32, G13, C70

Keywords: Asset substitution/risk incentive problem, signalling, tradeoff theory, optimal capital structure, continuous-time finance

1 Introduction

Corporate bonds might be issued with several different provisions giving the bondholders some additional contractual rights. One of these rights is a so-called step-up provision, which states that the initial coupon rate paid to the holders of the bond will be increased once some predefined event takes place. Most frequently this event is linked to the rating of the issuing firm. If the rating is downgraded to some contractually laid down level, the coupon rate will be increased by a certain fraction. A sizeable volume of such rating-trigger step-up bonds have been issued in particular by firms from the telecom industry (see table 1 for a representative example).

From an investors' perspective, such a provision might be considered as nice to have, since it promises a higher payment at a time when the credit risk of the firm increases and thus the bond price would suffer otherwise. However, from the perspective of the issuing firm, it is much less clear for why it might be a good idea to write such a contract because firms have to pay out more to its debt holders when less cash is available.

Empirical evidence for the consequences from a step-up feature is given by

Table 1: Deutsche Telekom's Debt Issuance Program (excerpt)

ISIN	Principal	Coupon	Maturity Date
XS0158875673	€ 500,000,000	6.123%*	Dec. 04, 2007
DE0006210446	€ 1,000,000,000	5.75%*	Feb. 12, 2008
XS0155788150	€ 500,000,000	6.5%*	Oct. 07, 2009
XS0155312829	GBP 500,000,000	7.125%*	Sep. 26, 2012
XS0158739739	GBP 250,000,000	7.375%*	Dec. 04, 2019

* In the event of ratings change by Moody's and S&P that causes the ratings to be below of Baa1 by Moody's and BBB+ by S&P the interest rates on the notes will increase by 0.5% with effect from the first interest payment date after this rating change occurs. (Reversible)

Houweling et al. (2004) and Lando and Mortensen (2004). The latter calibrate a reduced-form model and compare step-up bonds to otherwise similar straight fixed-coupon bonds. They find that step-up bonds increase the cost of capital for the issuer so that step-up features should be avoided. The observation that

firms still use step-up bonds seems to be even more puzzling, when we analyze this aspect in a typical tradeoff model for the optimal capital structure in a world with tax benefits and bankruptcy costs (see e.g. Fischer et al. (1989) and Leland (1994)). Within this modelling approach, the optimal debt volume, that maximizes the firm value, is positively related to the underlying state variable (e.g. asset value or firm's instantaneous cash flow). Hence, if the state variable deteriorates and the firm value declines, an increase of the debt obligation cannot be optimal in order to add firm value.[1]
As it is frequently the case in financial economics, two notorious distortions of the perfect markets assumption are considered to explain this apparent puzzle: Agency conflicts and asymmetric information. Thus, we have (at least) two intuitive candidate motives for the use of step-up provisions. The reasoning behind the first distortion is that the step-up feature might be able to mitigate the risk-shifting (asset substitution) incentive of manager-owners because a higher risk increases the likelihood of a rating-trigger. Regarding the second distortion, the step-up provision might also be considered as a credible device to signal some non-observable pricing-relevant firm characteristics to potential investors because primarily risky firms might not want to have a (costly) step-up feature.
Our paper is not the first to come up with these explanations for the use of step-up bonds. Bhanot and Mello (2006) address the asset substitution problem, while Manso et al. (2007) analyze the signalling hypothesis within the broader class of performance-sensitive debt contracts. The broad conclusion emerging from existing literature is that step-up bonds cannot solve the risk-shifting problem,[2] but are able to credibly signal non-observable firm characteristics.[3]

However, there are still a number of open issues and shortcomings, that leave unsettled the question if step-up bonds can be an optimal financing instrument. First, Bhanot and Mello (2006) do not address a general optimization problem. On the one hand, they do not consider the optimal step-up bond design as the result of maximizing ex ante firm value with respect to all verifiable character-

[1] Recent work by Strebulaev (2007) finds empirical evidence that data are more consistent with comparative static predictions from trade-off theory than is traditionally thought.

[2] "In general, an increase in the coupon level decreases firm value and does not inhibit (and might even stimulate) asset substitution."Bhanot and Mello (2006), Remark 6, p. 91.

[3] See Manso et al. (2007), Proposition 1, p. 21.

istics of such a contract.[4] On the other hand, they only allow for a restricted risk-shifting strategy.[5] Second, in the signaling game of Manso et al. (2007), firms can choose between issuing performance-sensitive debt or equity. They find that performance-sensitive debt can establish a separating equilibrium in cases where this is not possible with a straight bond.[6] However, when debt is a crucial financing instrument for firms (in order to benefit from tax benefits or to prevent the loss of control rights to new investors), a signaling game, where the signal is given by the specific bond design and not by the choice between debt and equity should be considered. Third, both contributions neither aim at characterizing the optimal step-up bond design, nor discuss conditions (with respect to the relevant parameters) under which the use of step-up bonds might be optimal. This however, are prerequisites to infer testable implications.

In this paper, we want to close the gap from the three concerns mentioned above. For this purpose, we treat both motives (separately) in a unified framework. Our goal is to characterize the optimal contract design as well as to derive testable implications about the use of step-up bonds.

We find that in contrast to Bhanot and Mello (2006), step-up bonds can mitigate the agency conflict if the more general optimization problem is solved. We are able to derive closed-form solutions for the optimal step-up bond design and can characterize the conditions under which the use of step-up bonds is optimal. With respect to asymmetric information, we show that similar to Manso et al. (2007), a separating equilibrium can be established when a signal is derived from the specific bond design (i.e. the inclusion or exclusion of a step-up provision). We can describe the optimal contract design and provide results concerning the conditions for an equilibrium. Our major finding is that the equilibrium predictions from the two hypotheses contrast sharply regarding the optimal bond design and the optimal use of step-up bonds. In particular, the firm characteristics and an observed bond design can immediately explain whether a risk-shifting problem or a problem of asymmetric information is the main reason for why a firm uses step-up bonds:

In terms of the optimal bond design, a bond with a finite step-up factor is is-

[4] The same shortcoming applies to related work by Silva and Pereira (2007).

[5] In their model, manager-owners have the possibility to alter the investment program only right after debt issuance. Later risk-shifts are not possible.

[6] This approach extends results by Ross (1977) to situations where bankruptcy costs are low.

sued given that the risk-shifting problem can be mitigated with step-up bonds. Conversely, for a problem of asymmetric information, a bond with a zero-coupon is issued to signal favorable information that starts paying a coupon (infinite step-up factor) once the rating event is triggered. Regarding the optimal use of step-up bonds, we obtain that firms with a high initial risk never use step-up bonds to mitigate risk-shifting problems, while for problems of asymmetric information step-up bonds can be an attractive device. In general, step-up bonds are primarily used when the risk-shifting problem is sufficiently severe, while for signalling reasons even a modest problem of asymmetric information supports the use of step-up bonds.

The remainder of the paper is organized as follows: The next section puts up the general model framework and establishes that absent any frictions, step-up bonds are not optimal. In section 3, we introduce agency conflicts in the sense that manager-owners might follow a self-interested risk-shifting policy. Section 4 considers the alternative explanation that the use of step-up bonds is due to asymmetric information problems. Finally, in section 5, the different equilibrium predictions are discussed. Section 6 concludes. Proofs are contained in the appendix.

2 General Model Framework

We consider a firm that owns some productive assets generating a continuous cash-flow x, whose dynamics are given by

$$\mathrm{d}x_t = \mu x_t \,\mathrm{d}t + \sigma x_t \,\mathrm{d}Z_t, \qquad x_0 > 0, \tag{1}$$

where, as usual, $\mathrm{d}Z_t$ denotes the increment of a standard Wiener process and μ and σ are constant parameters. For pricing purposes, we assume perfect capital markets on which a risk-free asset with a constant instantaneous risk-free interest rate r is continuously traded. Either all market participants are risk-neutral or markets are arbitrage-free which implies that there exists a martingale measure which allows for risk-neutral pricing. In the latter case μ denotes the risk-adjusted drift term which is restricted to $\mu < r$ to guarantee finite security values. The present value of any arbitrary claim $C(x)$ whose instantaneous payoff is an affine function $a\,x + b$ on the state variable x, can then be written as the sum of the present value of the flow of payoffs to the claimholders from time t up to some (stopping) time $\mathcal{T}$ and the present value of the claim at that time :

$$C_t = \mathbb{E}_t \left[\int_t^{\mathcal{T}} e^{-r(s-t)}(a\, x_s + b)\, \mathrm{d}s + e^{-r \cdot \mathcal{T}} \mathcal{L}(x_{\mathcal{T}}) \right].$$

To apply this general framework to the case of a firm that considers to issue debt with a rating-trigger step-up feature, note that such a contract will essentially consist of three elements:

(i) The initial coupon rate c before a rating-trigger,

(ii) the step-up factor $\delta > 1$ (i.e. after a step-up event has occurred, the new coupon is δc), and

(iii) the trigger threshold $x_T < x_0$, i.e. once the cash flow x hits the barrier x_T, the step-up takes place. We consider the case of an irreversible step-up event. If subsequent to the step-up, x rises above x_T again, the coupon rate remains at δc.

Thus, we can completely characterize the step-up bond contract by the triple (c, δ, x_T). The equity holders are residual claimants in the sense that they immediately receive the cash flow that exceeds the coupon obligations. If the cash flow is insufficient to cover the coupon payments, deep-pocketed equity holders have to make up for the difference. This standard payout policy means that we rule out that the firm finances the coupon payments by selling part of the assets and that the dividend is set strategically.[7] Furthermore, we assume that absolute priority of the debt claim is enforced so that renegotiations, which result in strategic debt service, cannot take place.[8] Since this rule implies that equity holders are left with nothing in the case of default, we obtain the following representation for the equity value, which we denote by S:

$$\begin{aligned} S_t &= \mathbb{E}_t \left[\int_t^{\mathcal{T}_b} e^{-r(s-t)}(1-\tau)(x_s - c - 1_{\{s \geq \mathcal{T}_T\}}(\delta - 1)c)\, \mathrm{d}s \right] \qquad (2) \\ &= \mathbb{E}_t \left[\int_t^{\mathcal{T}_T} e^{-r(s-t)}(1-\tau)(x_s - c)\, \mathrm{d}s + \int_{\mathcal{T}_T}^{\mathcal{T}_b} e^{-r(s-t)}(1-\tau)(x_s - \delta c)\, \mathrm{d}s \right], \end{aligned}$$

[7] See e.g. Morellec (2001) for a model where assets can be sold to finance debt service or dividend payments. Strebulaev (2007) allows for asset sales if firms enter into financial distress.

[8] See e.g. Anderson and Sundaresan (1996), Mella-Barral (1999), Fan and Sundaresan (2000), Koziol (2006) or Hackbarth et al. (2007) for the implications of strategic debt service.

where $1_{\{.\}}$ denotes the indicator function and τ stands for the tax rate on corporate income. For notational convenience, we abstract from further taxes on the personal level. $\mathcal{T}_b$ denotes the stopping time, i.e. the time of default of a levered firm, while $\mathcal{T}_T$ denotes the time when the rating-trigger level is attained.

To have a meaningful problem, we can focus on those step-up bonds that imply $\mathcal{T}_T < \mathcal{T}_b$, i.e. the step-up occurs before the firm defaults. It is apparent that a step-up bond design so that the coupon δc after a step-up will never be paid to debt holders but the firm defaults before, is not optimal to increase the value of a firm that uses a straight consol bond. Intuitively, this is due to the fact that for $\mathcal{T}_T \geq \mathcal{T}_b$ the coupon payments to debt holders are like those of a straight consol bond but the potential increase of the coupon obligation can result in an earlier costly default.

With standard pricing techniques, we can write the equity value S in (2) as of time t_0 in the following way:

$$\begin{aligned} S_0 &= (1-\tau)\Big[\left(\frac{x_0}{r-\mu}-\frac{c}{r}\right)-\left(\frac{x_b}{r-\mu}-\frac{c}{r}\right)\left(\frac{x_0}{x_b}\right)^{\beta} \\ &- \frac{(\delta-1)c}{r}\left(\left(\frac{x_0}{x_T}\right)^{\beta}-\left(\frac{x_0}{x_b}\right)^{\beta}\right)\Big], \end{aligned} \tag{3}$$

where x_b and x_T are the cash flow levels that determine the corresponding stopping times, i.e. $\mathcal{T}_b = \inf\{s; x_s = x_b\}$ and $\mathcal{T}_T = \inf\{s; x_s = x_T\}$. The parameter $\beta < 0$ can be obtained from the (negative) root of the characteristic equation $\frac{\sigma^2}{2}y(y-1)+\mu y - r = 0$ in y and amounts to

$$\beta = -\frac{\mu-\sigma^2/2+\sqrt{2\,r\,\sigma^2+(\mu-\sigma^2/2)^2}}{\sigma^2}.$$

The variable β contains the characteristic parameters μ and σ that drive the cash flow process and the risk-free rate r.

The term $\left(\frac{x}{x_{(\cdot)}}\right)^{\beta}$, that plays an important role for all security values, has the interpretation of a probability-weighted discount factor,[9] i.e. the present value of one unit of account that is paid out if and only if the process x_t hits the boundary $x_{(\cdot)}$ from above for the first time. Therefore $0 < \left(\frac{x}{x_{(\cdot)}}\right)^{\beta} \leq 1$ holds.

Equation (3) shows that the equity value with the step-up feature equals the

[9] This interpretation follows from the fact that $\mathbb{E}\left[\int_0^{\mathcal{T}_{(\cdot)}} e^{-r\,s}\,\mathrm{d}s\right] = \left(\frac{x_0}{x_{(\cdot)}}\right)^{\beta}$. See also Mella-Barral (1999), p. 541.

equity value under plain debt (first line) for the given default barrier x_b plus an additional term $(1-\tau)\left[\frac{(\delta-1)c}{r}\left(\left(\frac{x_0}{x_T}\right)^\beta-\left(\frac{x_0}{x_b}\right)^\beta\right)\right]$ which accounts for the additional coupon payments once the step-up has taken place, i.e. the trigger level x_T has been attained. Since $x_T \geq x_b$, this component is always positive (for $\delta > 1$) and thus reduces the equity value for a given default barrier x_b. We note that the optimal default barrier x_b depends on the coupon δc after a step-up.

Analogous reasoning leads to the debt value. However, due to the absolute priority rule, we need to specify the value of debt in case of default. In line with most of the literature,[10] we assume that in case of default the debt value equals the value of an unlevered firm minus bankruptcy costs. The variable α denotes the bankruptcy costs as a fraction of the unlevered firm value $(1-\tau)\frac{x_t}{r-\mu}$ at the default time. The default value $\mathcal{L}(x_t)$ at the cash flow level x_t is then given by

$$\mathcal{L}(x_t) = (1-\alpha)(1-\tau)\frac{x_t}{r-\mu}.$$

We note that a default either means a restructuring of the firm or a liquidation. As long as both events are associated with bankruptcy costs, it is not crucial for us which type of a default is present.

Applying the general solution, we obtain the following representation for the present value of debt D:

$$D_0 = \frac{c}{r} + \left(\mathcal{L}(x_b) - \frac{c}{r}\right)\left(\frac{x_0}{x_b}\right)^\beta + \frac{(\delta-1)c}{r}\left(\left(\frac{x_0}{x_T}\right)^\beta - \left(\frac{x_0}{x_b}\right)^\beta\right). \quad (4)$$

In line with the equity value S, we can understand the debt value D as the sum of two components. The first two terms correspond to the value of a straight bond for a given default barrier x_b. The last term $\frac{(\delta-1)c}{r}\left(\left(\frac{x_0}{x_T}\right)^\beta-\left(\frac{x_0}{x_b}\right)^\beta\right)$ captures the present value of an increase of the coupon due to a rating-trigger.

The value of the levered firm, which we denote by V is then the sum of (3) and (4):

$$V = S + D.$$

In general, the initial owners of the firm face the problem to design the step-up bond so that the firm value is maximized in t_0 given the initial cash flow level x_0. In our setup, this is done by choosing some optimal security design, i.e. by fixing the terms of the debt issue (c, δ, x_T). Note that these decision variables are

[10] See e.g. Goldstein et al. (2001), Morellec (2004) or Hackbarth et al. (2007).

contractible. Once the debt is issued, the firm acts in favor of the equity holders rather than the entire firm value. Thus, the default barrier x_b, which cannot be part of a contract, is chosen by the firm so that the equity value is optimized. As a consequence, the optimization problem of the firm reads:

$$\begin{aligned} &\max_{(c,\delta,x_T)} V(c,\delta,x_T,x_b^*) \\ &\text{s.t.} \\ &x_b^* = \arg\max_{x_b} E(c,\delta,x_T,x_b). \end{aligned} \tag{5}$$

Since a default can only take place after the step-up event, i.e. when the firm has straight debt with a coupon δc outstanding, we can also apply the well-known representation[11] for the optimal default barrier in the case of straight debt by incorporating the coupon δc *after* a step-up:

$$x_b = \delta c \cdot \frac{(r-\mu)}{r} \frac{\beta}{(\beta-1)}, \tag{6}$$

This barrier is a result of the smooth-pasting condition of the equity value S in the cash flow x_t.[12] The barrier x_b is linear in the coupon δc and independent of the actual cash flow level x_t.

Now, we can plug in the solution for the default constraint in order to obtain the optimal design of the step-up bond that maximizes the firm value. We find that bonds with a step-up feature are not required for an optimal firm value. We state this as

Proposition 1 *It is not optimal for a firm to issue a rating-trigger step-up bond, as long as there are no agency conflicts regarding risk-shifting and no problems of asymmetric information.*

It is easily verified that the derivative of V with respect to the trigger level x_T is given by

$$\frac{\partial V}{\partial x_T} = c\tau \frac{(\delta-1)(-\beta)}{r x_T} \left(\frac{x}{x_T}\right)^{\beta},$$

which is apparently positive for any choice of $c > 0$ and $\delta > 1$. Thus, a trigger barrier x_T below the current cash flow level x_0 is not optimal.

[11] See e.g. Goldstein et al. (2001) or Hackbarth et al. (2007).

[12] It can be shown that the result of the value-matching and smooth-pasting condition is equivalent to the maximization of the equity claim. See e.g. Dixit (1993) or Dixit and Pindyck (1994).

The intuition for why the step-up feature always destroys firm value, given no problems regarding risk-shifting and asymmetric information are present, is that the step-up bond increases the coupon obligation at a time when the firm generates lower cash flows and thus rather wants to decrease its debt burden than to increase it. As a consequence, the existence of step-up bonds cannot be explained within this basic set-up and we need to incorporate additional model features. As mentioned above, agency conflicts and asymmetric information might be motives for the optimal use of a step-up bond.

3 Agency Conflicts

3.1 Optimal Step-up Bond Design

In this section, we assume that the firm has the possibility to change the investment program. More precisely, in line with most of the literature on asset substitution,[13] we consider the case that the manager-owners of the firm have a unique, irreversible opportunity to alter the risk profile of the assets in place in the sense that the volatility of the cash flow process can be increased from σ to σ_H, while all other parameters remain unaffected.

In the case of straight debt, we know from the basic Leland (1994) model that a higher volatility σ_H ceteris paribus results in a lower firm value. However, from the perspective of the equity holders only, an increase of the risk is desirable because σ_H increases the equity value due to the option-like nature of their claim, or in more technical terms, due to the convexity in the state variable. This phenomenon is known as the risk incentive or asset substitution problem.

Absent any possibility for debt holders to discipline or to put sanctions on the manager-owners, the latter will immediately increase the risk after the debt is issued. This risk-shift, however, will be anticipated by the debt holders and thus the firm is only able to place its debt issue at the unfavorable high risk terms. Obviously, the firm would be better off and could add firm value, if this agency conflict could be mitigated. In what follows, we explore the capability of step-up bonds to resolve this conflict.

We formalize the idea of a risk-shift by introducing another threshold x_σ at

[13] See e.g. Leland (1998), Ericsson (2000) or Flor (2006).

which the manager-owners change the cash flow process from the low to the high risk. Apparently, if a firm has a rating-trigger step-up bond outstanding, the barrier x_σ at which the firm increases the risk must be above or equal to the rating-trigger barrier x_T. This is a consequence of the fact that after a debt issuance the firm acts in favor of the equity holders. Since the firm effectively has a straight bond with coupon δc outstanding once the cash flow hits x_T, the firm will definitely increase its risk at this barrier as long as it has not done so before. Therefore, we can restrict ourselves to the case $x_T \leq x_\sigma$. With analogous notation, we can express the equity value as

$$S_t = (1-\tau)\mathbb{E}_t\Big[\int_t^{\mathcal{T}_\sigma} e^{-r(s-t)}(x_s^\sigma - c)\,\mathrm{d}s \;+\; \int_{\mathcal{T}_\sigma}^{\mathcal{T}_T} e^{-r(s-t)}(x_s^{\sigma_H} - c)\,\mathrm{d}s \\ +\; \int_{\mathcal{T}_T}^{\mathcal{T}_b} e^{-r(s-t)}(x_s^{\sigma_H} - \delta c)\,\mathrm{d}s\Big],$$

where $\mathcal{T}_\sigma = \inf\{s; x_s = x_\sigma\}$ and the notation x^{σ_H} indicates that the higher diffusion parameter is involved. Evaluating the above expression and the corresponding expression for the debt value, we find

$$\begin{aligned} S_0 &= (1-\tau)\Big\{\left(\frac{x_0}{r-\mu}-\frac{c}{r}\right)-\left(\frac{x_b}{r-\mu}-\frac{c}{r}\right)\left(\frac{x_0}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_b}\right)^{\beta_H} - \\ &\quad \frac{(\delta-1)c}{r}\Big[\left(\frac{x_0}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_T}\right)^{\beta_H} - \left(\frac{x_0}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_b}\right)^{\beta_H}\Big]\Big\} \qquad (7) \\ D_0 &= \frac{c}{r} + \left(\mathcal{L}(x_b)-\frac{c}{r}\right)\left(\frac{x_0}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_b}\right)^{\beta_H} + \\ &\quad \frac{(\delta-1)c}{r}\Big[\left(\frac{x_0}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_T}\right)^{\beta_H} - \left(\frac{x_0}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_b}\right)^{\beta_H}\Big], \qquad (8) \end{aligned}$$

where β_H indicates that the high risk σ_H is involved. Note further that according to (6) the optimal default boundary depends on β. Since the risk-shift occurs before the default threshold is attained, x_b in the above expressions is determined using β_H rather than β.[14]

Note that it is a priori not clear whether $\left(\frac{x}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_b^H}\right)^{\beta_H}$ will be greater or smaller than $\left(\frac{x}{x_b}\right)^{\beta}$. On the one hand, due to the higher risk after a risk-shift, the threshold x_b^H will be attained faster. On the other hand, because the default threshold

[14] In order to be precise, we should write this as x_b^H. However, to ease notation, we omit the superscript if no explicit reference is needed.

is determined endogenously, x_b^H is lower than x_b, so that the net effect is a priori undetermined.

To determine the optimal design of a step-up bond in the presence of a risk-shifting possibility, the firm faces a similar optimization problem as in (5), i.e. the firm value is maximized with respect to the step-up bond features (c, δ, x_T). The difference is that not only the default barrier x_b is set by the firm in order to maximize the equity value after debt issuance but also the risk-shifting barrier x_σ, because a risk-shift is not contractible. The firm value follows from the sum of (7) and (8). These considerations result in the following optimization problem:

$$\begin{aligned} &\max_{(c,\delta,x_T)} V(c, \delta, x_T, x_b^*, x_\sigma^*) \\ &\text{s.t.} \\ &(x_b^*, x_\sigma^*) = \arg\max_{x_b, x_\sigma} E(c, \delta, x_T, x_b, x_\sigma). \end{aligned} \tag{9}$$

This formulation of the optimization problem differs in two important points from the model in Bhanot and Mello (2006): First, we address the problem of optimal security design, in the way that the step-up design must be the outcome of maximizing the firm value with respect to the triple (c, δ, x_T). From our perspective, this is the reasonable notion, since these bond characteristics are contractible. In contrast, Bhanot and Mello (2006) proceed in the following way. They fix the amount of debt raised through a bond without a step-up provision, and examine the effect on the equity value if the same amount of debt is raised through a debt issue that includes a step-up feature. This approach, however, cannot solve the optimal security design problem in general.[15]

Second, we allow for a risk-shift at an arbitrary point in time, i.e. we consider an endogenous risk-shifting policy by ex post equity holders. Since the risk-shifting policy is observable but not contractible (otherwise we would not have a meaningful agency conflict), it acts as a constraint to the optimization problem. In Bhanot and Mello (2006) the risk-shifting policy is exogenously restricted, and can only occur immediately after a bond issue.

To solve the optimization problem in (9), we already clarified the optimal solution of x_b in the previous section, so it remains to be shown how the risk-

[15] Work by Silva and Pereira (2007) suffers from a similar drawback. They only solve a one-dimensional maximization problem.

shifting barrier is optimally set. To this end, it is helpful to rewrite the equity value in (7) as follows:

$$\begin{aligned} S_0 &= (1-\tau)\left(\tfrac{x_0}{r-\mu}-\tfrac{c}{r}\right)-(1-\tau)\left(\tfrac{x_0}{x_\sigma}\right)^{\beta}\left(\tfrac{x_\sigma}{x_T}\right)^{\beta_H}\cdot Q, \\ &\text{with: } Q \equiv \left(\tfrac{c}{r}(\delta-1)+\left(\tfrac{x_T}{x_b}\right)^{\beta_H}\left(\tfrac{x_b}{r-\mu}-\tfrac{\delta c}{r}\right)\right). \end{aligned} \tag{10}$$

Note that Q is independent of the risk-shifting strategy x_σ. Thus, the maximization of the equity value S in x_σ is equivalent to the maximization (minimization) of the factor $\left(\frac{x}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_T}\right)^{\beta_H}$ given that Q is negative (positive). One can easily show that

$$\frac{\partial}{\partial x_\sigma}\left(\frac{x}{x_\sigma}\right)^{\beta}\left(\frac{x_\sigma}{x_T}\right)^{\beta_H} > 0$$

holds. Apparently, the earlier the firm increases the risk of the cash flow process (i.e. x_σ is higher), the earlier a given barrier x_T is hit. As a consequence, we can directly derive the optimal solution for x_σ as

$$x_\sigma = \begin{cases} x_0 & \text{if } Q < 0 \\ x_T & \text{if } Q > 0 \\ [x_T, x_0] & \text{if } Q = 0 \end{cases}. \tag{11}$$

The optimal risk-shifting policy is to switch to the more risky investment either instantaneously or to wait until the trigger threshold is attained. The result has an intuitive interpretation. The term Q is the sum of the present value of the additional coupon payment in perpetuity minus the value of the option to default conditional on arriving at the level x_T. If the absolute value of the option exceeds the value of the additional coupon payment then $Q < 0$ and it is optimal for equity holders to switch immediately to the high risk strategy. In that case the disadvantage from additional coupon payments is only moderate relative to the advantage given by the option to voluntarily default. On the other hand, if the value of the additional coupon payment exceeds the absolute option value, then $Q > 0$ and equity holders will find it optimal not to increase the risk until the trigger threshold x_T has been hit. Note that Q itself depends on the terms of the step-up bond (c, δ, x_T). For notational convenience, we will call bonds for which a risk-shift is not optimal before x_T, i.e. $Q > 0$, as bonds that satisfy the *risk mitigation property.*

We can always find a design of step-up bonds so that the risk mitigation property is satisfied, while also different designs exist for which the risk mitigation property does not hold. With a step-up factor δ close to one, the step-up property is not very pronounced and the firm has no incentive to prevent an increase of the risk to avoid additional coupon payments. Formula (6) for the optimal default barrier confirms that in this case Q is always negative. Conversely, if the step-up factor δ and the trigger barrier x_T are sufficiently high, the step-up feature is very severe. Thus, it is plausible that the firm wants to prevent a step-up trigger which implies no voluntary risk-shift. In this case, Q is positive because the factor $\left(\frac{x_T}{x_b}\right)^{\beta_H}$ tends to zero but the first term $\frac{c}{r}(\delta - 1)$ is very large. We summarize these findings as

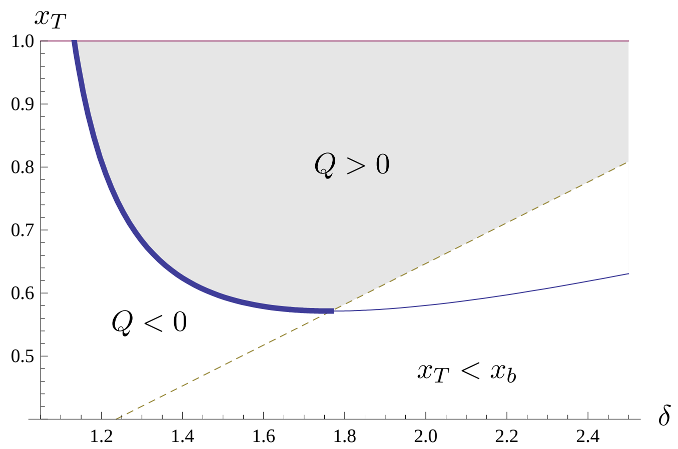
Proposition 2 *(Endogenous risk-shifting policy) If a firm has an arbitrary rating-trigger step-up bond outstanding and manager-owners have a unique irreversible possibility to change the risk from σ to $\sigma_H > \sigma$, they will either increase the risk promptly or wait until the cash flow process x hits the rating-trigger barrier x_T. Other risk-shifting strategies x_σ with $x_T < x_\sigma < x_0$ are never optimal.*

If the asset substitution problem is to be solved by the issue of a step-up bond, proposition 2 tells us that the terms of the bond have to be set such that $Q \geq 0$. At $Q = 0$, where the manager-owners are indifferent with respect to the timing of the risk-shift, we assume that they act in favor of the firm value so that x_σ also equals x_T.

Figure 1 visualizes the part of the (δ, x_T)-space (given some arbitrary coupon c) for which Q is positive or negative. The dashed line indicates all trigger barriers x_T which are equal to the default threshold x_b. Thus, this line excludes the region on the right side from this line because for those pairs (δ, x_T) the step-up feature is not relevant as the default barrier x_b is above the trigger barrier x_T. In the region below the convex solid curve, the risk mitigation property is not satisfied, i.e. the pairs (δ, x_T) imply $Q < 0$. In other words, for a given x_T, the step-up factors δ are too low to provide a sufficient incentive for the firm to postpone a risk-shift. Accordingly, for a given δ, the x_T in that region are too high to be incentive compatible.
Consequently, the remaining shaded region contains all feasible, incentive compatible combinations of x_T and δ, which might be candidates for the optimal design of the step-up bond. Appendix A shows that the optimal design of a step-up bond must consist of pairs (δ, x_T) from the boundary indicated by the bold section of the convex curve. An intuitive explanation for the fact that there is no interior solution in the shaded region is as follows: Since the step-up of the

Figure 1: Critical Step-Up Barrier $x_T(\delta)$ and Risk Mitigation Area

The diagram shows the combinations (δ, x_T), which satisfy the risk mitigation property, i.e. $Q \geq 0$ holds, as the grey shaded area. The other combinations (δ, x_T) either violate the risk mitigation property (x_T is below the convex function $x_T(\delta)$ for those δ) or the trigger barrier x_T would exceed the default barrier x_b (x_T is below the dashed line for those δ) which contradicts a reasonable step-up design. The other parameter values are: $x_0 = 1$, $c = 2$, $\sigma = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.15$, $\tau = 0.35$, $r = 0.07$, and $\mu = 0.05$.



coupon has a negative effect on the firm value, as long as the riskiness of the assets is given, the firm wants to keep the step-up factor on a level that is as low as possible for a given trigger barrier x_T which implies $Q = 0$. From this equality we can deduce a relation between a minimum x_T and δ that satisfies the risk mitigation property, i.e.

$$x_T(\delta) = x_b \left(\frac{c(\mu - r)(\delta - 1)}{r\, x_b + c(\mu - r)\delta} \right)^{1/\beta_H} = x_b(c, \delta) \left((\beta_H - 1)(1 - \delta)\delta^{-1} \right)^{1/\beta_H} \qquad (12)$$

Note that the trigger threshold is a multiple of the default threshold x_b which itself is a function of c and δ.

Figure 1 also shows that the relation between the minimum x_T and δ is negative, i.e. for a given combination of x_T and δ, a lower x_T can only be achieved by increasing δ. This is plausible, since a lower x_T means that the risk mitigation property is supposed to apply for a longer time. Thus, the step-up feature δ, that prevents the risk-shift, must be more pronounced.

Equation (12) provides the solution to the incentive constraint, and is key to determining the optimal firm value. Let us denote by $\mathcal{S}$ the set of pairs (δ, x_T) that lie on the bold section of the graph $x_T(\delta)$ in figure 1. $\mathcal{S}$ is characterized by a minimum and a maximum step-up factor δ. A δ above the minimum step-up factor δ_{min} ensures that the step-up factor is high enough so that the firm has no incentive to increase the risk before the cash flow x hits the barrier x_T. A δ below the maximum step-up factor δ_{max} implies that a risk-shift (and accordingly a step-up) in fact takes place before the firm defaults. We can define the set $\mathcal{S}$ as follows:

$$\mathcal{S} = \{(\delta, x_T); \forall\, \delta' \in (\delta_{min}, \delta_{max}) :\ x_T = x_T(\delta')\} \tag{13}$$

where δ_{min} is the smallest δ such that $x_T \leq x_0$:

$$\delta_{min} = \min\{\delta'; x_T(\delta') \leq x_0\}$$

and δ_{max} is the highest δ such that $x_T \geq x_b$:

$$\delta_{max} = \max\{\delta'; x_T(\delta') \geq x_b\}.$$

We will give a precise characterization of δ_{min} and δ_{max} when analyzing whether a step-up bond is worthwhile for a firm or not.

The important property that a step-up bond can add firm value, *if and only if* the solution (δ^*, x_T^*) to the maximization problem lies within the set $\mathcal{S}$, reduces the problem to two dimensions: the coupon c and the step-up factor δ. This is due to the fact that the choice of the step-up factor δ uniquely determines the trigger barrier $x_T(\delta)$. The remaining determination of the solution is conceptually straightforward but algebraically tedious and follows from first- and second-order conditions. Appendix B contains the corresponding details. The optimal step-up design is characterized by the following closed-form representations, which we summarize in

Proposition 3 *(Optimal design) Given that a step-up bond is optimal to mitigate the asset substitution problem, the optimal design* (c^*, δ^*, x_T^*) *of a rating-trigger step-up bond is the solution to the optimization program (9) and given by the following closed-form formulae:*

$$c^* = \frac{(\beta-1)}{\beta}\frac{r}{(r-\mu)}\left(1-\frac{\beta_H}{\beta}\right)^{-1/\beta_H} x_T^*, \tag{14}$$

$$\delta^* = \frac{\beta\,(\beta_H-1)}{\beta_H\,(\beta-1)}, \tag{15}$$

$$x_T^* = x_0\left(\left(\frac{\alpha}{\tau}-\alpha+1\right)(\beta_H-\beta)\right)^{1/\beta}. \tag{16}$$

Proposition 6 in the next section will give a formal characterization for which firms a step-up bond is optimal. Note, that plugging in c^* and δ^* in (6) yields the optimal default threshold

$$x_b^* \;=\; \left(1 - \frac{\beta_H}{\beta}\right)^{-1/\beta_H} x_T^*. \tag{17}$$

The optimal firm value as of time $t = 0$ simplifies to

$$V_0^* \;=\; \frac{(1-\tau)x_0}{r-\mu} + \frac{\tau}{r-\mu}\, x_b^*. \tag{18}$$

The representation for the optimal firm value — given that a step-up provision is optimal — allows for a remarkable interpretation. This is because the representation for the firm value as in (18) also applies to the case of straight debt, the only difference being that the default barrier x_b^* has a different size. Interestingly, in the case of straight debt the default barrier, given as

$$x_{b,plain}^* = x_0 \left(\beta\left(\alpha - \frac{\alpha}{\tau} - 1\right)\right)^{1/\beta},$$

is lower than the default barrier in the case of a step-up. Therefore, we can economically interpret the cost from the risk-shifting possibility as a loss of the firm value which is directly revealed by a different default barrier. Moreover, it is well-known that a firm having an infinitely high risk σ_H cannot add firm value with a straight bond (see e.g. Leland (1994)) so that the optimal firm value equals the value $(1-\tau)\frac{x_0}{r-\mu}$ of an unlevered firm. The formulae for the optimal default barrier $x_{b,plain}^*$ together with the representation for the optimal firm value V_0^* confirm this effect, because β tends to zero for an infinitely high risk. In the case that the firm uses a step-up bond and can increase its risk σ_H infinitely high, i.e. $\beta_H \to 0$, the optimal default barrier x_b^*, however, is strictly positive

$$\lim_{\beta_H \to 0} x_b^* = x_0 \left(e \cdot \left(\frac{\alpha}{\tau} - \alpha + 1\right)(-\beta)\right)^{1/\beta}.$$

Thus, the optimal firm value V_0^* with a step-up bond exceeds the firm value with a straight bond even in this extreme situation where the firm can increase its risk arbitrarily high.

As a result of the closed-form representations for the optimal design of the step-up provision δ^* and x_T^*, we can derive the following testable implications, which we state as

Corollary 4 *If a firm optimally uses the step-up provision to counter the risk-shifting problem, a more pronounced risk-shifting problem, i.e. a higher* σ_H*, results in a higher step-up factor* δ^* *and a lower trigger barrier* x_T^**.*

This claim directly follows from the derivatives of (15) and (16) for β_H taking into account that β is negative and a strictly increasing function in σ. The intuition for this implication is as follows: If the risk-shifting problem is more pronounced, the firm wants to postpone the risk-shift for a longer time which results in a lower optimal trigger barrier x_T^*. To ensure that a risk-shift can in fact be prevented, a higher step-up factor is required in order to provide the equity holders with the incentive to keep the risk on the lower level σ.
From the optimal triple (c^*, δ^*, x_T^*), it is a priori not clear whether the optimal step-up design can mitigate the agency costs so strongly that it increases the firm value relative to a straight bond. The step-up feature is attractive for the firm if and only if the optimal bond design (c^*, δ^*, x_T^*) results in a feasible relationship between the default barrier x_b^*, the trigger barrier x_T^*, and the initial cash flow x_0, i.e. $x_b^* < x_T^* < x_0$. This relationship is equivalent to the condition that the step-up factor δ lies in the interval $(\delta_{min}, \delta_{max})$.
In what follows, we determine the conditions for which δ lies within the interval $(\delta_{min}, \delta_{max})$ and thus, a step-up bond adds firm value.

Turning to δ_{max} first, note that from (12), $x_T(\delta)$ is a multiple of $x_b(\delta)$, where the multiple depends on δ and β_H and can be larger or smaller than one. However, a multiple smaller than one would imply that $x_T < x_b$ which is not consistent with our definition of a step-up bond. Thus, we can deduce a maximum for δ which satisfies the risk mitigation property and results in an optimal trigger threshold $x_T \geq x_b$, denoted by δ_{max} as

$$\delta_{max} = \frac{\beta_H - 1}{\beta_H}.$$

Note that δ_{max} is determined by the intersection of the dashed graph with the solid line in figure 1 and is independent of c.[16] Comparing δ_{max} with the optimal δ^*, we can write

$$\delta^* = \frac{\beta\,(\beta_H - 1)}{\beta_H\,(\beta - 1)} = \frac{\beta}{(\beta - 1)}\,\delta_{max} < \delta_{max}.$$

[16] It can also be shown that the function $x_T(\delta)$ has its minimum in δ_{max}. This property further justifies the notion of a maximal δ, since although a higher δ is feasible, it is not possible to thereby decrease x_T.

Therefore, the upper boundary is never violated in the optimum. Alternatively, the solution for the optimal default barrier (17) indicates that the default barrier cannot exceed the optimal trigger barrier because the term $\left(1 - \frac{\beta_H}{\beta}\right)^{-1/\beta_H}$ is always below one. This technical property has an important economic interpretation and we highlight this result as

Corollary 5 *Every firm that optimally uses a step-up bond does not fully exclude a risk-shift but admits a region $(x_b^*; x_T^*)$ with positive length in which the risk of the firm's assets is high.*

Next, turn to δ_{min}. In general, this should be the solution of $x_T(\delta) = x_0$ in δ with $x_T(\delta)$ given as in equation (12). Unfortunately, this equation cannot be evaluated algebraically for an arbitrary coupon c. However, since we are only interested in the *optimal* triple (c^*, δ^*, x_T^*), we can plug in the optimal coupon $c^*(\delta)$ given any δ in (12). The corresponding equation has an algebraic solution given by

$$\delta_{min} = \frac{\beta_H(\beta - 1)(\alpha\tau - \alpha - \tau) - \tau}{\beta_H(\beta - 1)(\alpha\tau - \alpha - \tau)}.$$

In general, as long as the the optimal step-up factor is above δ_{min}, the optimal bond design is more favorable than a straight bond. While we could establish that the inequality $\delta^* < \delta_{max}$ for the upper barrier always holds, the inequality $\delta^* > \delta_{min}$ does not need to be valid in general. We can write δ_{min} as

$$\delta_{min} = \delta^* \cdot \frac{\beta_H(\beta - 1)(\alpha\tau - \alpha - \tau) - \tau}{\beta(\beta_H - 1)(\alpha\tau - \alpha - \tau)},$$

from which we can deduce an equivalent condition for $\delta_{min} < \delta^*$. This condition simplifies to

$$\beta_H - \beta > \frac{\tau}{(\alpha + \tau(1 - \alpha))}. \tag{19}$$

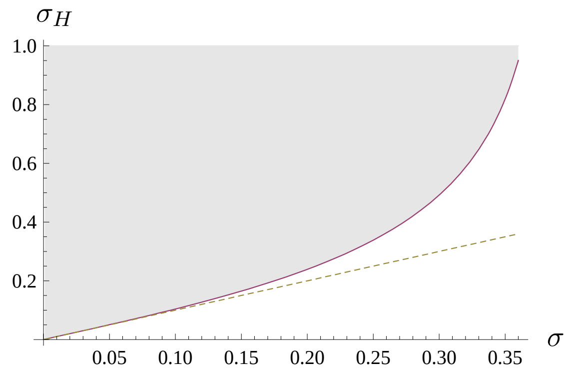
Since the inequality involves all relevant parameters, it completely characterizes the conditions under which a step-up provision can mitigate the agency conflict in the sense that it increases firm value. These fundamental findings are summarized in the next proposition.

Proposition 6 *(Optimal Use of Step-Up Bonds for Risk-Shifting) Suppose that a firm has an initial investment risk of σ and a unique, irreversible opportunity to increase the risk to σ_H, which is observable but not contractible. An optimally designed step-up bond can add firm value relative to a straight bond, if and only if condition (19) holds.*

It is instructive to see for which pairs (σ, σ_H) a step-up feature is worthwhile for the firm, i.e. to determine the conditions on (σ, σ_H) under which it is value-enhancing to solve the agency conflict through issuing a step-up bond. For this purpose, we have to translate condition (19) into a corresponding relation for the risk parameters. Figure 2 indicates when a step-up provision is attractive. The convex function $\bar{\sigma}_H(\sigma)$ refers to the case that condition (19) holds with equality and is drawn as the solid graph. Since $\beta_H - \beta$ strictly increases with σ_H, condition (19) is always satisfied for $\sigma_H > \bar{\sigma}_H(\sigma)$. Thus, for all risk levels σ_H above the critical level $\bar{\sigma}_H(\sigma)$, a step-up feature is attractive for the firm, while for risk levels $\sigma_H < \bar{\sigma}_H(\sigma)$ it is not. Thus, the shaded region contains all pairs (σ, σ_H) for which a step-up bond is able to mitigate the agency problem.[17]

Figure 2: Optimal Use of Step-Up Bonds

The diagram shows the combinations (σ, σ_H) as grey shaded area, for which firms optimally issue step-up bonds. The dotted line $\sigma_H = \sigma$ indicates the minimum value for the feasible high risk σ_H The other parameter values are: $\alpha = 0.15$, $\tau = 0.35$, $r = 0.07$, and $\mu = 0.05$.



The first important outcome from this figure is that for firms with a sufficiently low initial risk σ, a step-up bond is always worthwhile as long as the possibility to (at least slightly) increase the risk $\sigma_H - \sigma > 0$ exists. Interestingly, as the initial risk σ is increased, the minimum risk $\bar{\sigma}_H(\sigma)$ after a risk-shift for which a step-up bond is still attractive increases more than proportionally. An intuitive explanation for this result is as follows: The value of a firm with straight debt is a convex and declining function in the business risk σ. Thus, a risk-shift primarily

[17] The dashed straight line indicates $\sigma_H = \sigma$. Obviously, the region below this line is not relevant, since $\sigma > \sigma_H$ is not consistent with the notion of asset substitution.

hurts firms with a low initial risk σ. For this reason, a step-up feature, that prevents a risk-shift, is especially chosen by low-risk firms.
It can even be shown that there exists a limit for σ beyond which step-up bonds are *never* optimal. To formally prove this, note that $\beta(\sigma) < 0$ is monotonically increasing in σ towards a limit equal to zero. Therefore, the maximum initial β such that condition (19) is satisfied, is given by the critical value $\overline{\beta}$:

$$\overline{\beta} = -\frac{\tau}{(\alpha + \tau(1 - \alpha))},$$

which in turn determines the maximum σ. This critical value follows from equation (19) with $\beta_H = 0$. In line with intuition, once the initial risk σ is very high, it is not worthwhile anymore to implement any step-up feature. This is because the potential to prevent a firm value decline due to a risk-shift is relatively low but the costs in form of a loss of the firm value from the step-up feature are still present. We summarize these findings as

Corollary 7 *Firms can increase the firm value with a step-up bond relative to the use of a straight bond in the presence of a risk-shifting possibility in two cases: (i) The initial risk of the firm is sufficiently low. (ii) The risk-shifting possibility, $\sigma_H - \sigma$, is very pronounced and the initial risk σ is below a threshold $\overline{\sigma}$. Conversely, if the initial risk is too high, i.e. σ exceeds $\overline{\sigma}$, then a step-up bond is never worthwhile for the firm independent of the risk-shifting option.*

3.2 Illustration of Optimal Step-up Features

In this section we illustrate the results with a numerical example. Since the firm value and the step-up features are homogenous of order one in x_0, we normalize $x_0 = 1$ without loss of generality. For the parameters, we choose a base case scenario of: $\mu = 0.05$, $r = 0.07$, $\alpha = 0.15$ and $\tau = 0.35$ which is broadly consistent with previous literature.[18] Suppose the initial investment risk is $\sigma = 0.2$ and the high-risk investment is $\sigma_H = 0.3$.

Table 2 and figure 3 show the numerical results. If the firm issues plain debt

[18] See e.g. Goldstein et al. (2001), Huang and Huang (2002), Morellec (2004), Hackbarth et al. (2007) and Bhanot and Mello (2006). The parameter choices are close to Bhanot and Mello (2006) to enable direct comparison, except for α, i.e. bankruptcy costs. While we choose $\alpha = 0.15$, which is broadly consistent with empirical evidence according to Andrade and Kaplan (1998) or more recently Strebulaev (2007), proportional bankruptcy costs in Bhanot and Mello (2006) amount to 60%.

Table 2: Optimal Firm Values and Contract Design

The table reports the optimal firm value and the corresponding bond design for a straight bond with low risk $\sigma = 0.2$ and high risk $\sigma_H = 0.3$. As a further financing possibility a step-up bond is considered where the firm has the possibility to shift its risk from σ to σ_H. The other parameter values are: $x_0 = 1$, $\alpha = 0.15$, $\tau = 0.35$, $r = 0.07$, and $\mu = 0.05$.

Bond type	c^*	δ^*	x_T^*	x_b^*	Firm value	Agency costs
Plain (low risk)	2.76	-	-	0.58	42.63	-
Plain (high risk)	2.91	-	-	0.47	40.75	1.88
Step-up (risk-shift)	2.33	1.29	0.79	0.48	41.06	1.57

and were able to contract upon the risk choice, the optimal firm value is 42.63 representing the first-best solution. Since the risk policy is not contractible, investors anticipate the incentive of manager-owners to switch to the high risk opportunity immediately after the issue of plain debt. Therefore, in the case of a plain debt issue, the optimal firm value is only 40.75 as a result of a risk-shift initiated by the owner-managers. The associated agency costs from the risk-shifting problem can be measured as the difference between the optimal firm value in the first-best case without risk-shifting possibility and the optimal firm value in the case for the high risk σ_H. In the example, the agency costs amount to 1.88. With the issuance of an optimal step-up bond, it is possible to reduce these agency costs to 1.57, which is a reduction of roughly 16.5%. The optimal step-up factor turns out to be 1.29, i.e. an increase in the coupon rate of 29% at a trigger level of 0.79.

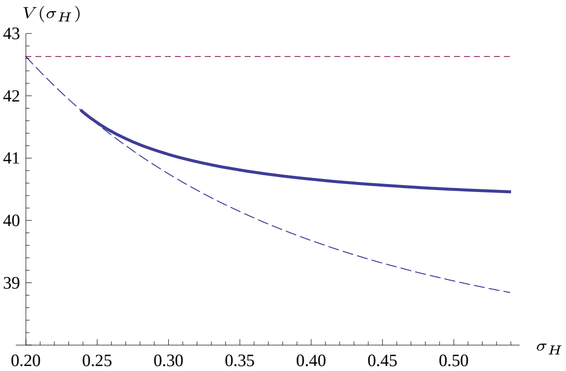
Figure 3 shows the firm value for different financing cases as a function of the high-risk opportunity σ_H. In particular, this graph indicates the optimal value of a firm with straight debt without the risk-shifting possibility $V_{plain(\sigma)}$, the optimal value of a firm with straight debt $V_{plain(\sigma_H)}$ with the risk-shifting possibility, and the optimal value of a firm with a step-up bond. This graph indicates the agency costs of the risk-shifting problem as the difference

$$V_{plain(\sigma)} - \max\left(V_{plain(\sigma_H)}, V_{step-up}\right)$$

between the optimal firm value without the risk-shifting possibility and in the presence of the risk-shifting possibility. If the risk-shifting problem is present, the optimal firm value is either the value $V_{plain(\sigma_H)}$ of a firm with straight debt

Figure 3: Optimal Firm Values for Different Financing Possibilities

The diagram shows three optimal firm values as a function of a high risk σ_H. The first firm value (dotted line) $V_{plain(\sigma)}$ refers to a firm that uses straight debt and cannot alter its risk. In the second case, the firm can increase its risk to σ_H so that $V_{plain(\sigma_H)}$ (dashed line) is the optimal firm value of a firm with straight debt and high risk. The third firm value $V_{step-up}$ (solid line) is the result from an optimal financing with a step-up bond where a risk-shifting possibility from σ to σ_H is given. The other parameter values are: $x_0 = 1$, $\sigma = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.15$, $\tau = 0.35$, $r = 0.07$, and $\mu = 0.05$.



and high risk or the optimal firm value $V_{step-up}$ with a step-up bond. As figure 3 reveals, higher values for σ_H (at a constant $\sigma = 0.2$) imply higher agency costs and can be interpreted as a more pronounced agency conflict. The step-up bond is able to achieve a proportionally higher mitigation of agency costs, when the agency conflict becomes more severe. In this numerical example, a step-up bond is not optimal for σ_H below $\bar{\sigma}_H = 0.239$, at which point the solid line ($V_{step-up}$) smooth-pastes to the dashed line ($V_{plain(\sigma_H)}$).

4 Asymmetric Information

In the previous section, we have analyzed the possibility of step-up bonds to mitigate agency conflicts. Now, in this section, we turn to the competing hypothesis for the use of rating-trigger step-up bonds which is the presence of asymmetric information problems. Therefore, we will analyze the possibility for firms to use step-up bonds to convey information about the true business risk to the capital market. Since we know from the previous section that a step-up feature is primarily painful for high-risk firms, a step-up feature might be an attractive device for a low-risk firm to credibly distinguish itself from a high-risk firm. Given that a separating equilibrium exists, we want to figure out the optimal design of the step-up feature. This, in turn, will enable us to identify remarkable differences relative to the optimal step-up bond design when agency problems are present.

For our analysis we consider two types of firms, each having a cash-flow process following the same dynamics as described in (1), except that type L has diffusion parameter σ, while the type H firm has a diffusion parameter σ_H with $\sigma_H > \sigma$. Thus type L is a low-risk firm, while H is the high-risk firm.[19] The potential investors know that every firm can be of type L or H, but they have per se no indication about a firm's true type. The owner-managers of the initially non-levered firm, however, know the true type which causes a typical problem of asymmetric information. Since we focus on the ability of step-up bonds to signal the true type, we abstract from further incentive conflicts such as the risk-shifting possibility addressed in the previous section. After an issue of a step-up bond, the owner-managers obtain the proceeds from the debt issue in form of a special dividend and they might still hold the equity of a now levered firm. To determine the firm value or even more precisely the wealth obtained by the initial owner-managers, the use of the equity position is crucial. In general, the two alternatives are to hold the shares or to sell them. We can state right from the outset that in the extreme case that the owner-managers sell their shares to outsiders immediately after the debt issuance, a separating equilibrium cannot exist. Apparently, in this case the wealth obtained by the owner-managers equals the equity and the debt value according to the perception of the capital market. Assume that a bond design for good firms existed that could signal a more favorable firm value so that the sum of the values of equity and debt (according to the market perception) increased. Then every other arbitrarily bad firm could also choose this design and the owner-managers would obtain the same wealth as the owner-managers of the good firm. Therefore, every firm could mimic a good firm so that a credible bond design that signals favorable information about the firm does not exist. The formal reason for this finding is the fact that the

[19] Manso et al. (2007) construct the information asymmetry with respect to the drift term μ of the diffusion process. Thus, in their signaling game, a high-productivity type tries to separate itself from a low-productivity type. Since the crucial impact on firm value is due to the β factor, whereby β is an increasing function in σ and a decreasing function in μ, both formulations of the information asymmetry are qualitatively equivalent.

Spence-Mirrlees condition does not hold, which requires that the marginal costs for sending a signal differ between the two types.[20] This is clearly violated because the owner-managers obtain the same payoff irrespective of the firm's true type.[21]

However, in the case that some of the shares are kept by the owner-managers, the step-up feature might be able to act as a signalling device. To analyze this important characteristic of step-up bonds, we assume for simplicity that the owner-managers hold their stocks infinitely long so that the market perception is not relevant for the equity value.[22]

We will use the following notational convention: $C_{i,j}^m$ denotes a claim (firm value or debt value) on the cash-flow of a firm that is actually of type $i \in (L, H)$, but which is perceived by outsiders as being of type $j \in (L, H)$, while offering a contract (i.e. sending the message) $m \in \{(c, \delta, x_T); c \geq 0, \delta \geq 1, x_b < x_T \leq x_0\}$. Note that $m = (c, 1, x_0)$ describes a plain bond, while we indicate a (yet undetermined optimal) step-up bond by $m = (c, \delta, x_T)$.[23] Since the relevant equity value held by the owner-managers is evaluated with the knowledge of the true type, the equity value E_i^m will carry only one subscript which refers to the true type of the firm. Like for the other claims, the superscript m indicates the bond design. For example, $D_{H,L}^{(c,\delta,x_T)}$ denotes the debt value of a high-risk firm being perceived as a low-risk firm that has issued a step-up bond. The firm value from the perspective of the owner-managers is

$$V_{i,j}^m = E_i^m + D_{i,j}^m.$$

This objective indicates that for a given market perception j, the true type of the firm still matters as the equity value E_i^m is driven by the true type i rather than the market perception j, while the debt position $D_{i,j}^m$ is priced according to the market perception j. Therefore, the costs for a low-risk firm from issuing a step-up bond differ from those of a high-risk firm (and hence, the Spence-Mirrlees condition is fulfilled).

---

[20] For the Spence-Mirrlees conditions, see e.g. Bolton and Dewatripont (2005).

[21] The above reasoning is consistent with results by Nachman and Noe (1994) who find that a (plain) debt contract is optimal to minimize mispricing losses if conditions are such that separating equilibria cannot exist.

[22] Manso et al. (2007) consider traded equity. However, they assume that the information asymmetry is dissipated immediately after securities are issued. ("The growth rate becomes public knowledge after the firm raises capital." Manso et al. (2007), p. 20.)

[23] Note that a continuum of possible messages exist, since the only restrictions on (c, δ, x_T) are given by $c \geq 0, \delta \geq 1, x_b < x_T \leq x_0$.

To analyze the outcome of a signalling game, we apply the concept of a perfect Bayesian equilibrium which is mainly characterized by the fact that posterior beliefs are determined through Bayes' rule. We denote beliefs about the true type $i \in (L, H)$, conditional on the information content from the choice (c, δ, x_T) of the bond contract, as $\pi(i|(c, \delta, x_T))$. The signals, i.e. the bond contract characteristics, are chosen such that they maximize the objective function of the informed party. Posterior beliefs about out-of-equilibrium actions are not restricted and can take on any value in $[0, 1]$. The objective function of the informed party is the expected firm value:

$$\mathbb{E}[V_i^{(c,\delta,x_T)}] = \pi(L|(c, \delta, x_T)) \cdot V_{i,L}^{(c,\delta,x_T)} + \pi(H|(c, \delta, x_T)) \cdot V_{i,H}^{(c,\delta,x_T)}.$$

In general, possible equilibria in a signalling game can be classified into separating, pooling, or semi-separating equilibria, and it is well-known that the kind of equilibrium depends on the actual conditional beliefs of the uninformed party. Since it is our focus to analyze whether step-up bonds are a device to credibly signal the firm's true type, we restrict our interest to separating equilibria of the signalling game. Thus, it is natural to consider a situation, where each type of firm chooses a *different* message in equilibrium. Therefore, we consider the following posterior beliefs:

$$\pi(L|(c, \delta, x_T)) = 1, \qquad \pi(H|(c, \delta, x_T)) = 0.$$

If the uninformed party receives the message (c, δ, x_T), with $\delta > 1$ and $x_b < x_T < x_0$, i.e. it observes that the firm offers a step-up bond, she beliefs the firm to be of type L. For any other contract design (c', δ', x'_T), beliefs are such that the uninformed party considers the firm to be of type H, i.e.

$$\pi(L|(c', \delta', x'_T)) = 0, \qquad \pi(H|(c', \delta', x'_T)) = 1.$$

From the proof of proposition 1, we know that a step-up feature is always costly in the sense that it decreases the firm value. Thus, if the H type considers to choose a different step-up design than (c, δ, x_T), only a plain consol bond can ever be optimal, i.e. $(c', \delta', x'_T) = (c, 1, x_0)$, and we can restrict our attention to the situation where only two contracts are offered.

A separating equilibrium is then characterized by the fact that by taking into account this posterior beliefs, a firm actually issues a step-up bond only if it is

type L, while the type H firm actually prefers straight debt, so that the beliefs are therefore justified.
The posterior beliefs, as specified above, are intuitively plausible. The step-up feature is always costly in the sense that it decreases the firm value. For a L type firm, it might be worthwhile to take on this cost in order to be recognized as such, while the H type firm will only have an incentive to mimic the behavior of the L type as long as the benefit of more favorable debt terms will outweigh the costs associated to the step-up provision. As long as this net benefit is positive, the H type firm will mimic the issuing behavior of the L type firm and benefit from the mispricing gain. Only a step-up design such that this net benefit turns negative can be considered as a candidate equilibrium outcome. Therefore we must impose the incentive compatibility condition

$$V_{H,H}^{(c,1,x_0)} \geq V_{H,L}^{(c,\delta,x_T)}$$

for the H type, and likewise

$$V_{L,L}^{(c,\delta,x_T)} \geq V_{L,H}^{(c,1,x_0)}$$

for the L type. Since the issuance of plain debt will be considered as a signal that the firm is of type H, the coupon rate in that case is determined through the parameter σ_H, i.e. $(c^*(\sigma_H), 1, x_0)$.

In general, the incentive compatibility constraints determine the set of separating equilibria, and this set might contain many of them. From the perspective of the L type firm, it is natural to consider the problem to find some optimal equilibria, where optimality in our setup refers to the maximization of the firm value (or more precisely, the wealth obtained by the former firm-owners). In this sense, we can summarize the optimal step-up bond design problem under asymmetric information as the following optimization program:

$$\begin{aligned} &\max_{\{c,\,\delta,\,x_T\}} \left(V_{L,L}^{(c,\delta,x_T)} \right) \\ &\text{s.t.} \\ &\begin{cases} V_{L,L}^{(c,\delta,x_T)} \geq V_{L,H}^{(c,1,x_0)} \\ V_{H,H}^{(c,1,x_0)} \geq V_{H,L}^{(c,\delta,x_T)} \end{cases} \end{aligned} \tag{20}$$

The L type firm tries to achieve the highest possible firm value, conditional on the fact that a separating equilibrium can be achieved, where the design of the bond issue perfectly reveals the true type.[24] Therefore, the incentive compatibility inequalities act as constraints to the maximization problem.[25] If a solution (c, δ, x_T) with $\delta > 1$ does not exist, the L type firm issues a plain bond and is perceived as being of type H.

Whenever an equilibrium exists, the incentive condition for the H type firm must be binding. Otherwise, the design of the step-up bond could be (marginally) changed in a neighborhood around (c, δ, x_T) so that the incentive condition of the H type firm is still valid. In particular, a higher step-up barrier x_T can be implemented. Since a higher x_T increases the firm value $V_{L,L}^{(c,\delta,x_T)}$ of the L type firm due to higher tax benefits, the initial step-up bond design (c, δ, x_T) before the increase of x_T cannot be optimal.

An analysis of the optimization program brings up two problems: (i) Under which conditions do separating equilibria exist. (ii) Given that these conditions are fulfilled, i.e. a separating equilibrium exists, what is the optimal design of (c, δ, x_T). In general, a separating equilibrium can exist but does not need to exist. In particular, under asymmetric information, we obtain the following result for the use of step-up bonds that contrasts remarkably the equilibrium predictions under the asset substitution hypothesis. We state this in the following proposition. The proof is in appendix C.

Proposition 8 *(Optimal Use of Step-Up Bonds for Asymmetric Information) For any arbitrary risk σ of the low risk firm, any corresponding risk σ_H, with $\sigma_H \in (\sigma, \sigma + \epsilon)$ for a positive and sufficiently small ϵ, allows for the determination of a separating equilibrium, i.e. the incentive compatibility conditions in (20) are fulfilled.*

This proposition has two important consequences. First, if there is a marginal

[24] Note that this formulation of the problem aims not only at establishing a separating equilibrium (or a set of equilibria) but already tries to single out a specific equilibrium of the signalling game. Since the market is aware of the optimizing behavior of the firm, this puts restrictions on their out-of-equilibrium beliefs. Actually, an equilibrium solution of the program (20) would satisfy, or would be the one that survives the Cho-Kreps intuitive criterion (see Cho and Kreps (1987)). This refinement selects a unique pure-strategy equilibrium, that represents the *least-cost* separating equilibrium.

[25] To be precise, we should also note that, as in the previous section, the condition $x_b^* = \arg\max_{x_b} E_L^{(c,\delta,x_T)}$ is a further constraint to the maximization problem, which we left out to ease notation.

information asymmetry in the sense that σ_H is only infinitesimally higher than σ, a separating equilibrium, in which a step-up bond perfectly reveals the true type, is possible. Second, this proposition also implies that there does *not* exist an upper boundary on σ beyond which a separating equilibrium is never possible, regardless of σ_H.
The next section further analyzes the opposing equilibrium predictions for the design and feasibility of step-up bonds under the case of an agency conflict and asymmetric information.

In the remainder of this section, we will deal with the second aspect namely the optimal bond design in a separating equilibrium. Assume that the conditions for a separating equilibrium are fulfilled, i.e. the incentive compatibility constraints are satisfied, then we want to determine which choice of the triple (c, δ, x_T) maximizes the firm value of type L. In principle, one might proceed as follows: Since we know that the incentive compatibility constraint of the H type binds in equilibrium, we can take this as an equality, solve it for c, δ, or x_T, and plug in the solution in $V_{L,L}^{(c,\delta,x_T)}$. However, there does not exist an algebraic solution to $V_{H,H}^{(c,1,x_0)} = V_{H,L}^{(c,\delta,x_T)}$ in (c, δ, x_T), since different exponents β, β_H are involved additively. Therefore, we cannot supply closed-form solutions in general. However, we are able to derive an important qualitative result about the optimal equilibrium step-up bond design. To state this result in the next proposition, it is helpful to introduce the following simple notational convention. Denote the coupon rate *after* a step-up has taken place as $\hat{c}$. Obviously, this is defined as $\hat{c} \equiv \delta\, c$.[26]

Proposition 9 *The optimal design of a step-up bond under the condition that a separating equilibrium exists, i.e. such that the incentive compatibility constraints in (20) are satisfied, must be such that* $c \to 0$, $\delta \to \infty$ *and* $0 < \delta c = \hat{c} < \infty$.

The proof (which is given in appendix D) relies on an application of the implicit function theorem and shows that for any $\hat{c}$ the substitution rate $\frac{\mathrm{d}c}{\mathrm{d}x_T}$, that ensures a constant firm value, is smaller for the type L firm than that for the type H firm. To see how this leads to the stated result, consider some arbitrary combination $(c, \hat{c}, x_T)$ such that $V_{H,H}^{(c,1,x_0)} \geq V_{H,L}^{(c,\delta,x_T)}$ binds with equality. Then for any $\hat{c}$ an increase in x_T must be compensated by a reduction in c to keep the incentive compatibility constraint binding. Because $\frac{\mathrm{d}c}{\mathrm{d}x_T}$ is smaller for type L than for type H, this change of x_T and c increases the firm value for L. Since

[26] Likewise, the notation (c, δ, x_T) is equivalent to $(c, \hat{c}, x_T)$. A plain bond might be indicated by $(c, 1, x_0)$, or $(0, c, x_0)$.

this property holds for all initial values of c and x_T, the optimal choice is such that $c \to 0$ and x_T is chosen as high as possible.
This result is remarkable, since in comparison to the optimal design of the step-up bond in the presence of the asset substitution problem, it provides an entirely different equilibrium prediction. We will discuss these aspects in more detail in section 5.
Table 3 and figure 4 illustrate the optimal step-up design under asymmetric information by numerical optimization results. As in section 3 we consider the base case scenario with $\mu = 0.05$, $r = 0.07$, $\alpha = 0.15$, $\tau = 0.35$, and $\sigma = 0.2$. The first row in table 3 corresponds to the first-best solution, i.e. the L type

Table 3: Firm Values for Different Types of Financing and Market Perceptions

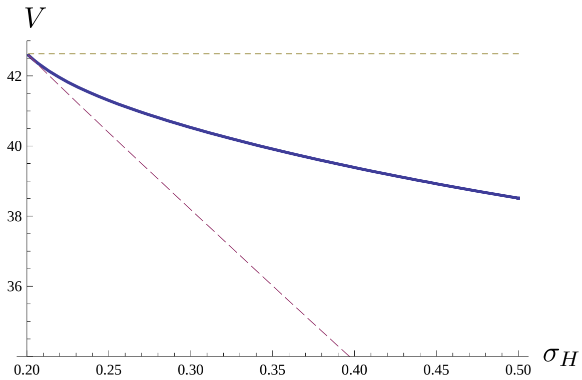
The table reports firm values, the corresponding bond contracts, and the costs of asymmetric information when firms use straight debt $(c^*(\sigma), 1, x_0)$ *and step-up bonds* $(c^*, \hat{c}^*, x_T^*)$. *The parameter values are:* $x_0 = 1$, $\sigma = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.15$, $\tau = 0.35$, $r = 0.07$, *and* $\mu = 0.05$.

Scenario	c^*	$\hat{c}^*$	x_T^*	x_b^*	Firm value	AI costs
$V_{L,L}^{(c^*(\sigma),1,x_0)}$	2.76	-	-	0.58	42.63	-
$V_{L,H}^{(c^*(\sigma_H),1,x_0)}$	2.91	-	-	0.61	38.19	4.44
$V_{L,L}^{(c^*,\hat{c}^*,x_T^*)}$	$\to 0$	2.45	0.94	0.51	40.53	2.1
$V_{H,H}^{(c^*(\sigma_H),1,x_0)}$	2.91	-	-	0.47	40.75	0

firm is recognized as such and issues plain debt ($V_{L,L}^{(c^*(\sigma),1,x_0)}$). However, under asymmetric information this firm value is not achievable, since investors belief that plain debt will be issued by H type firms. Given these beliefs, the second row reports the corresponding results for $V_{L,H}^{(c^*(\sigma_H),1,x_0)}$. The costs of asymmetric information (AI) amount to more than 10% of the initial firm value. The third row ($V_{L,L}^{(c^*,\hat{c}^*,x_T^*)}$) reports results for the firm value of the L type where a separating equilibrium is established with a step-up bond. As proven in proposition 9, the optimal initial coupon rate goes to zero, while the step-up factor approaches infinity such that the optimal coupon rate after the step-up ($\hat{c}^*$) assumes a finite value. The optimal step-up threshold x_T^* is chosen as high as possible so that the incentive compatibility constraints still bind. Through this step-up design, the type L firm is able to reduce the AI costs by more than 50%. In the last row of table 3, results for $V_{H,H}^{(c^*(\sigma_H),1,x_0)}$ are shown. Although the L type firm achieves

Figure 4: Firm Value Alternatives for an L Type Firm

The diagram shows the firm value $V_{L,L}^{(c^*,\hat{c}^*,x_T^*)}$ *(solid line) obtained with a step-up bond and the value* $V_{L,H}^{(c^*(\sigma_H),1,x_0)}$ *(dashed line) an L type firm obtains with a straight bond as a function of the risk* σ_H *of the H type firm. The horizontal dotted line indicates the first-best solution* $V_{L,L}^{(c^*(\sigma),1,x_0)}$ *(which is independent of* σ_H*). The other parameter values are:* $x_0 = 1$, $\sigma = 0.2$, $\alpha = 0.15$, $\tau = 0.35$, $r = 0.07$, *and* $\mu = 0.05$.



a separating equilibrium and can improve its firm value relative to its outside option $V_{L,H}^{(c^*(\sigma_H),1,x_0)}$, the H type firm value is higher. This is a result which is not uncommon for the outcome of signalling games.[27]

Finally, we can see from figure 4 that even for a very small asymmetric information problem in the sense that σ_H is only marginally higher than σ, a step-up bond can add firm value in a separating equilibrium. This is in contrast to the results from section 3 regarding the asset substitution hypothesis.

5 Discussion of Equilibrium Predictions

In contrast to Bhanot and Mello (2006), but in accordance with Manso et al. (2007), we find that step-up bonds can be a device to mitigate the asset substitution problem, as well as to signal favorable firm characteristics. In order to distinguish which of the two explanations is consistent with empirical evidence, and to deduce potentially testable hypotheses, it is instructive to compare the corresponding equilibrium predictions. Such a comparison can be made with re-

[27] Koziol (2007) obtains a similar result when outside collateral is used to mitigate problems of asymmetric information.

spect to two aspects: (i) The contract design if a step-up bond is optimal, and (ii) the conditions under which a step-up design is optimal.

Consider aspect (i) first. Table 4 summarizes the equilibrium predictions concerning the optimal step-up design, i.e. the optimal choice of the triple (c, δ, x_T). If agency problems are present, the discussion in section 3 has shown that the

Table 4: Summary of Equilibrium Predictions

Bond design	Asset substitution	Signalling
c	> 0	$\to 0$
δ	Lowest possible δ (for given x_T)	$\delta \to \infty$ but finite $\hat{c} > 0$
x_T	Lowest possible x_T (for given δ)	Highest possible x_T so that incentive conditions hold

step-up bond is optimally designed in a way that the risk-shifting incentives can be mitigated. In particular, from (10) and figure 1 we know that this is optimally accomplished if $Q(\delta, x_T) = 0$ holds. As shown in appendix A, the optimal bond design requires a δ as small as possible for any choice of x_T. Likewise, it is optimal to choose a x_T as low as possible, for any δ, so that $Q = 0$ still holds. This is intuitively plausible. Since the step-up threshold determines the time when a risk-shift occurs, the firm wants to set this as low as possible. Thus, the optimal decision about (δ, x_T) balances the benefit from the risk mitigation property to the associated costs of the step-up feature.

In contrast, if asymmetric information problems are present, we obtain a completely different optimal step-up design. To achieve a separating equilibrium, the step-up bond design must be sufficiently costly for the high risk firm such that it has no incentive to mimic. The step-up design can be costly in different ways, and proposition 9 has established that the least-cost separating equilibrium from the viewpoint of the low risk firm (type L) is achieved by setting a step-up threshold as high as possible, reducing the initial coupon rate to zero, and implementing a step-up factor that is infinitely high, such that the coupon rate after the step-up assumes some optimal level.

The intuition for this contrasting optimal step-up design is the following. In the case of agency problems, the benefit from the step-up bond is to prevent a risk-shift. The time when a risk-shift occurs is intimately linked to the step-up threshold. Therefore, a step-up threshold as low as possible is optimal. In contrast, for signalling reasons, the benefit from the step-up bond is to achieve a separating equilibrium. This can be obtained by imposing costs on the potentially mimicking firm. The same amount of costs can be imposed by very different choices of (c, δ, x_T). In particular, there is no need to have a low level of x_T as in the asset substitution case. Actually, as discussed above, it is optimal to have x_T as high as possible, while reducing the initial coupon rate to zero. In other words, a reason for the contrasting step-up design is that in the case of agency problems the friction occurs *within* the firm, while for asymmetric information, the friction occurs *between* two different firms.

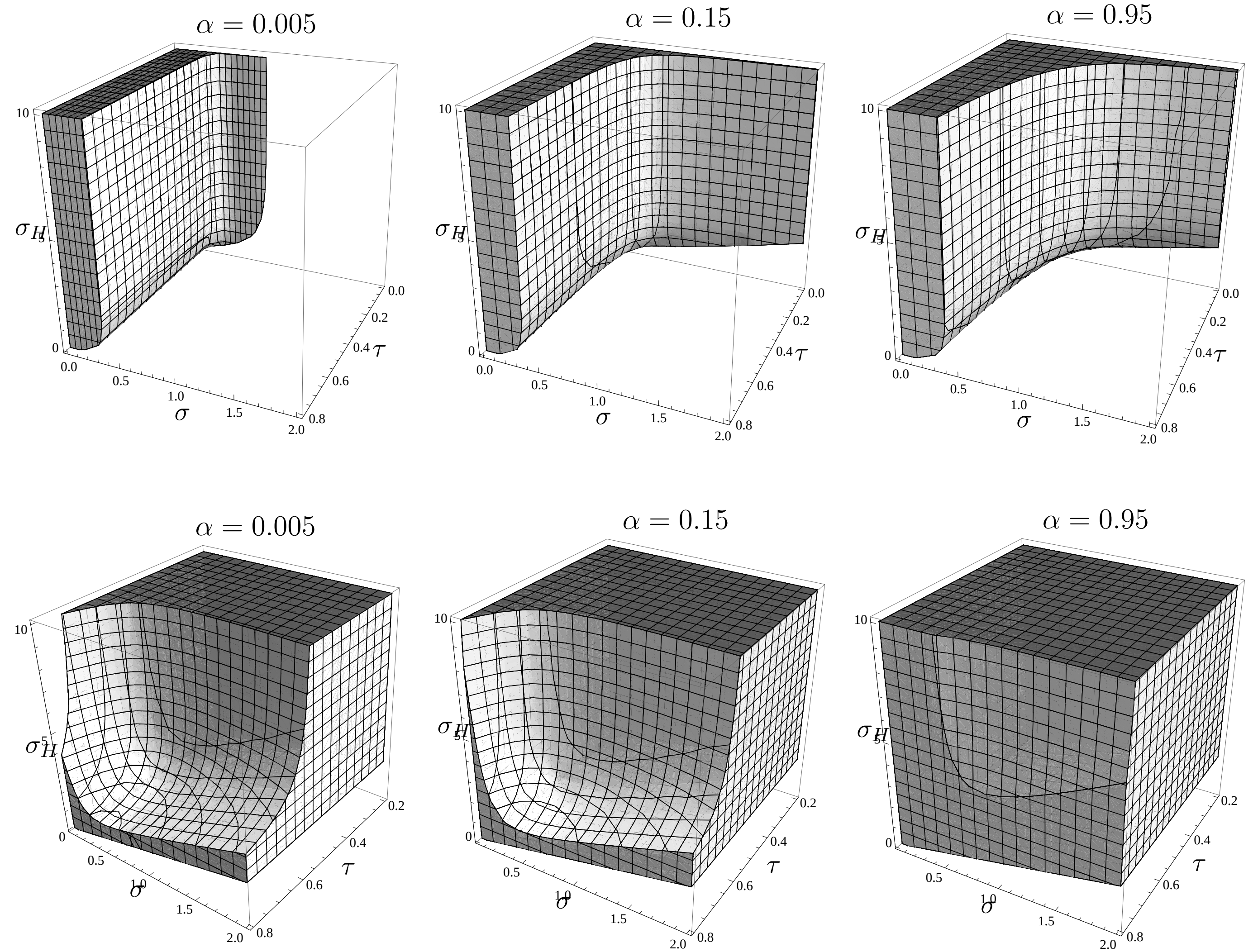
This outcome enables us to draw important conclusions concerning the use of step-up bonds as observed by empirical evidence. The example in the introduction (see table 1) as well as data in Lando and Mortensen (2004) and Houweling et al. (2004) show that the step-up in the coupon rate is on the order of 50 bp, or roughly 10% of the initial coupon rate. Thus, even without carrying out statistical tests, the empirical observations of the design of step-up bonds rather favors the agency conflict hypothesis than the asymmetric information hypothesis.

In what follows, we regard aspect (ii) in order to understand the conditions under which a step-up bond is worthwhile to mitigate risk-shifting versus asymmetric information problems. The crucial parameters, that determine if step-up bonds are optimal are the tax rate τ, the proportional bankruptcy costs α, and the risk parameters σ and σ_H. In figure 5, we indicate parameter constellations for which step-up bonds are feasible as the solid areas in the three-dimensional plot. The three panels in the upper row refer to the case of agency conflicts, while the three panels in the lower row refer to asymmetric information problems.[28] In line with results from section 3, we obtain from the upper row that for all volatility combinations (σ, σ_H) with $\sigma < \sigma_H$ a step-up bond can add firm value when the initial risk σ is not too high and additionally the risk σ_H after the risk-shift is sufficiently high (see proposition 6 and corollary 7). Moreover, we can observe that a lower tax rate τ and higher bankruptcy costs α increase the space in which a step-up bond is attractive for the firm. The equivalent represen-

[28] Thus, for the upper row, the plots are a 3-dimensional version of figure 2, which also contain the tax rate τ as a further dimension.

Figure 5: Admissible parameter values: Risk-Shifting Versus Asymmetric Information

The diagrams in the first line show combinations (τ, σ, σ_H) *for which a step-up bond is superior to straight bonds given that a risk shifting problem is present. The three diagrams refer to low bankruptcy costs* ($\alpha = 0.005$)*, normal bankruptcy costs* ($\alpha = 0.15$)*, and high bankruptcy costs* ($\alpha = 0.95$)*. The second line provides the corresponding diagrams for a problem of asymmetric information. The other parameter values are:* $x_0 = 1$*,* $r = 0.07$*, and* $\mu = 0.05$*.*



tation in the lower row shows that a separating equilibrium exists (for all other parameter values) whenever the tax rate is sufficiently close to zero. For tax rates above a critical value, no separating equilibrium exists when the volatility σ_H of the high-risk firm is very large. However, a separating equilibrium always exists whenever the risk σ_H of the high-risk firm only slightly exceeds the risk σ of the low risk firm. Moreover, a comparison of the three diagrams reveals that higher bankruptcy costs strongly increase the space in which a separating equilibrium exists.

A striking similarity between the optimal use of step-up bonds in the case of an informational problem and a risk-shifting problem is that step-up bonds are primarily chosen by firms which have high bankruptcy costs α and a low tax rate τ. Thus we can observe that those firms, which exhibit especially unattractive characteristics for the issuance of consol bonds (high bankruptcy costs and low present value of tax shields), use step-up bonds. Moreover, when the firm's risk σ is low, this effect is even fortified because — as illustrated above — a lower σ results in a higher firm value and therefore in a higher firm value increase due to a step-up feature.

Furthermore, there are two important differences between the optimal use of step-up bonds under informational and risk-shifting problems: The first difference refers to the level of σ. When the low risk σ is relatively high, step-up bonds are not attractive for firms that suffer from a risk-shifting problem. However, in the case of asymmetric information, a step-up bond can be an optimal financing instrument for low-risk firms with an arbitrarily high risk σ. This is due to the fact that a separating equilibrium exists for any σ whenever the risk σ_H of a high-risk firm does not exceed σ too much. Consequently, the observation that a firm with a high risk uses step-up bonds favors the assertion that this firm wants to signal that it does not have an even higher risk σ_H, because the opportunity to prevent a risk-shift cannot be worthwhile for it.

The second major difference concerns the level of σ_H. In the presence of an information problem, a lower σ_H for a given σ increases the attractiveness of step-up bonds. This is because if for a combination (σ, σ_H) a separating equilibrium with step-up bonds exist, then it is still optimal for any lower risk σ_H with $\sigma < \sigma_H$ to use a step-up bond. This is not necessarily true for higher σ_H. In the case of a risk-shifting problem, we obtain the opposite relationship, i.e. if for a combination (σ, σ_H) a step-up bond is optimally used, then for every higher risk σ_H a step-up bond is still optimal. Conversely, for a σ_H that only marginally exceeds the low risk level σ a step-up feature is not optimal in the case of a risk-shifting problem. As a consequence, in the case of an asset substitution problem, step-up bonds are only used when the risk-shifting problem is sufficiently severe, i.e. the difference $\sigma_H - \sigma$ is high, while for signalling reasons, even a modest information problem (i.e. the difference $\sigma_H - \sigma$ is low) supports the use of a step-up bond.

6 Conclusion

The observation that firms have recently issued rating-trigger step-up bonds brings up the question: What is the reason for a firm to commit itself to increase the debt service at a time when financial conditions have worsened? This question is even more puzzling in the framework of the well-established tradeoff models for the optimal capital structure. Two frictions, that are notorious in financial economics, might serve as explanations: Agency conflicts and asymmetric information problems. Results from the existing literature have discarded the former, but confirmed the latter hypothesis.

Our analysis contributes to this debate in the following points: First, by solving a more general optimization problem, which consists of maximizing firm value with respect to all (three) verifiable parts of the contract and which involves an optimal endogenous risk-shifting policy, we are able to provide closed-form solutions to the optimal step-up design when agency conflicts are present. Furthermore, we derive conditions under which it is optimal to solve the agency conflict by the inclusion of a step-up provision. We find that rating-trigger step-up bonds are indeed able to mitigate the agency conflict. Second, in line with existing results, we find that step-up bonds can establish a separating equilibrium and thus overcome asymmetric information problems. We are able to provide an important result about the optimal design, which then shows that the equilibrium predictions between the two hypotheses differ remarkably: A finite step-up factor, whose order of magnitude is consistent with observed step-up bonds, is optimal for bonds under a risk-shifting problem, while the optimal step-up factor in the case of asymmetric information is found to be infinite. Third, the opposing conditions for the characteristics of firms that issue step-up bonds to mitigate either agency conflicts or problems of asymmetric information directly allow to identify the underlying reason of a firm to use step-up bonds. A firm uses step-up bonds to mitigate the risk-shifting problem when the business risk can be increased considerably and/or the initial business risk is sufficiently low. Conversely, firms use step-up bonds to signal favorable information even when the problem of asymmetric information is only modest and irrespective of the initial business risk. An empirical test of these implications is left for further research.

A Proof of Relation (12)

In the text, it was shown that the solution to the optimal risk shifting threshold is

$$x_\sigma^* = \begin{cases} x_0 & \text{if } Q < 0 \\ x_T & \text{if } Q > 0 \\ [x_T, x_0] & \text{if } Q = 0 \end{cases} .$$

In principle any combination of (δ, x_T) that satisfies the inequality $Q > 0$ has the risk mitigation property and might therefore be a candidate for the optimal solution. Denote the set of pairs that satisfy the risk mitigation property as $\mathcal{M} = \{(\delta, x_T); Q > 0, x_b \leq x_T \leq x_0\}$. It is shown in the following that there does not exist an interior maximum for the firm value in $\mathcal{M}$.

For this it is enough to check for a given δ whether there exists a maximum in x_T. Write the firm value as

$$V(\delta, x_T) = A_1 + \left(\frac{x_0}{x_T}\right)^\beta \left(A_2 + A_3 \left(\frac{x_T}{x_b}\right)^{\beta_H}\right)$$

where:

$$A_1 = \frac{(1-\tau)x_0}{r-\mu} + \frac{\tau c}{r}, \quad A_2 = \frac{\tau(\delta-1)c}{r}, \quad A_3 = -\frac{\tau\delta c}{r} - \frac{\alpha(1-\tau)x_b}{r-\mu}$$

The derivative of V with respect to x_T reads

$$\frac{\partial V}{\partial x_T} = \left(\frac{x_0}{x_T}\right)^\beta \frac{A_3(\beta_H - \beta) - A_2\beta}{x_T} \left(\frac{x_T}{x_b}\right)^{\beta_H}$$

from which we can deduce the necessary condition

$$x_T^* = x_b \left(-\frac{A_2\beta}{A_3(\beta - \beta_H)}\right)^{1/\beta_H}$$

This optimum, however, is a minimum as can be verified by the sufficient condition:

$$\frac{\partial^2 V}{\partial x_T^2} = \left(\frac{x_0}{x_T}\right)^\beta \frac{\left[A_2\beta(1+\beta) + A_3(\beta - \beta_H)(1 - \beta_H + \beta)\left(\frac{x_T}{x_b}\right)^{\beta_H}\right]}{x_T^2}$$

The sign of the second derivative depends on the term in squared brackets. Plugging in x_T^* into this term yields a simplified representation equal to $A_2\,\beta\,\beta_H$ which is obviously positive for all parameter values. So, we can conclude that

$$\left.\frac{\partial^2 V}{\partial x_T^2}\right|_{x_T = x_T^*} > 0,$$

i.e. there does not exist a maximum in $\mathcal{M}$.

B Derivation of (c^*, δ^*, x_T^*) Under the Asset Substitution Hypothesis

The thresholds x_b and x_T can be defined as

$$\begin{aligned} x_b(\delta, c) &= \delta\, c\, \frac{(r-\mu)\beta_H}{r(\beta_H - 1)} \\ x_T(\delta, c) &= c\, \frac{(r-\mu)\beta_H}{r(\beta_H - 1)} \left((\beta_H - 1)(1-\delta)\delta^{\beta_H - 1}\right)^{1/\beta_H} \end{aligned}$$

Note that both are linear in c, which allows for the additional definitions

$$X_b(\delta) \equiv x_b(\delta, c)/c, \qquad X_T(\delta) \equiv x_b(\delta, c)/c.$$

We can write the firm value

$$\begin{aligned} V(\cdot) &= \left(\frac{(1-\tau)x_0}{r-\mu} + \frac{\tau c}{r}\right) - \left(\frac{\alpha(1-\tau)x_b}{r-\mu} + \frac{\tau c}{r}\right)\left(\frac{x_0}{x_T}\right)^{\beta_L}\left(\frac{x_T}{x_b}\right)^{\beta_H} + \\ & \frac{\tau(\delta-1)c}{r}\left(\frac{x_0}{x_T}\right)^{\beta_L}\left[1 - \left(\frac{x_T}{x_b}\right)^{\beta_H}\right] \end{aligned}$$

as a function of c

$$V(c) = \frac{(1-\tau)x_0}{r-\mu} + \frac{\tau c}{r} - c^{1-\beta_L}\left(K_1 - K_2\right), \tag{21}$$

where the functions

$$\begin{aligned} K_1(\delta) &= \left(\frac{\alpha(1-\tau)\delta\,\beta_H}{r(\beta_H - 1)} + \frac{\tau}{r}\right)\left(\frac{x_0}{X_T}\right)^{\beta_L}\left(\frac{X_T}{X_b}\right)^{\beta_H} \\ K_2(\delta) &= \frac{\tau(\delta-1)}{r}\left(\frac{x_0}{X_T}\right)^{\beta_L}\left(1 - \left(\frac{X_T}{X_b}\right)^{\beta_H}\right) \end{aligned}$$

which depend on δ but are invariant to c.

Differentiating with respect to c yields

$$\begin{aligned} \frac{\partial V(c)}{\partial c} &= \frac{\tau}{r} - (1-\beta_L)c^{-\beta_L}\left(K_1 - K_2\right) \\ c^* &= \left(\frac{(1-\beta_L)(K1 - K2)\, r}{\tau}\right)^{1/\beta_L} \end{aligned} \tag{22}$$

Substituting back (22) into (21) and simplifying gives

$$V(\delta) = \frac{(1-\tau)x_0}{r-\mu} + \frac{\tau\beta_L}{r(\beta_L-1)}\left(\frac{(1-\beta_L)(K_1(\delta)-K_2(\delta))\,r}{\tau}\right)^{1/\beta_L}$$

With the definitions of K_1 and K_2, we obtain

$$V(\delta) = \frac{(1-\tau)x_0}{r-\mu} + \frac{\tau\beta_L}{r(\beta_L-1)}\Bigg[\frac{\beta_H(\beta_L-1)(\alpha-\alpha\tau+\tau)(\delta-1)}{\tau} \left(\frac{x_0\,r(\beta_H-1)\left((\beta_H-1)(\delta-1)\delta^{\beta_H-1}\right)^{-1/\beta_H}}{\beta_H(r-\mu)}\right)^{\beta_L}\Bigg]^{1/\beta_L}$$

Differentiating this term for δ results in

$$\begin{aligned}\frac{\partial V(\delta)}{\partial\delta} &= \frac{\beta_H\delta-\beta_L(1+\beta_H(\delta-1))}{\beta_H(\beta_L-1)r(\delta-1)\delta}\Bigg[\frac{\beta_H(\beta_L-1)(\alpha-\alpha\tau+\tau)(\delta-1)}{\tau} \\ &\quad \left(\frac{x_0\,r(\beta_H-1)\left((\beta_H-1)(\delta-1)\delta^{\beta_H-1}\right)^{-1/\beta_H}}{\beta_H(r-\mu)}\right)^{\beta_L}\Bigg]^{1/\beta_L} \\ \delta^* &= \frac{\beta_L\,(\beta_H-1)}{\beta_H\,(\beta_L-1)} \qquad (23)\end{aligned}$$

Existence of this solution can be verified by substituting δ^* into the numerator of the first fraction in the above derivative.

$$\beta_H\delta^* - \beta_L(1+\beta_H(\delta^*-1)) = 0$$

For the given parameter restrictions, the following relations can be established and verify the uniqueness of this solution

$$\begin{aligned}\left.\frac{\partial^2 V(\delta)}{\partial\delta^2}\right|_{\delta=\delta^*} &< 0 \\ \left.\frac{\partial V(\delta)}{\partial\delta}\right|_{\delta<\delta^*} &> 0 \\ \left.\frac{\partial V(\delta)}{\partial\delta}\right|_{\delta>\delta^*} &< 0\end{aligned}$$

I.e. in δ^*, the second derivative is negative, giving the sufficient condition for a maximum. For $\delta < \delta^*$, the first derivative is positive, while for $\delta > \delta^*$ it is negative, verifying that there is no other optimum.

The optimal solution δ^* can be plugged in $c^*(\delta)$ and $c^*(\delta)$ can be plugged in $x_b(\delta, c(\delta))$ and $x_T(\delta, c(\delta))$. It can be shown that this gives the following optimal expressions:

$$
\begin{aligned}
x_T^* &= x_0 \left(\frac{(\alpha - \alpha\tau + \tau)(\beta_H - \beta_L)}{\tau} \right)^{1/\beta_L} \\
x_b^* &= \left(1 - \frac{\beta_H}{\beta_L}\right)^{-1/\beta_H} x_0 \left(\frac{(\alpha - \alpha\tau + \tau)(\beta_H - \beta_L)}{\tau} \right)^{1/\beta_L} \\
&= \left(1 - \frac{\beta_H}{\beta_L}\right)^{-1/\beta_H} x_T^* \\
c^* &= \frac{(\beta_L - 1)}{\beta_L} \frac{r}{(r-\mu)} \left(1 - \frac{\beta_H}{\beta_L}\right)^{-1/\beta_H} x_0 \left(\frac{(\alpha - \alpha\tau + \tau)(\beta_H - \beta_L)}{\tau} \right)^{1/\beta_L} \\
&= \frac{(\beta_L - 1)}{\beta_L} \frac{r}{(r-\mu)} x_b^*
\end{aligned}
$$

From these considerations, it follows that the optimal values of the firm, the debt and the equity are

$$
\begin{aligned}
V^* &= \frac{(1-\tau)x_0}{r-\mu} + \frac{\tau}{r-\mu} x_b^* \\
E^* &= \frac{(1-\tau)x_0}{r-\mu} - \frac{(1-\tau)(\beta_L - 1)}{(r-\mu)\beta_L} x_b^* \\
D^* &= \frac{(\beta_L - 1 + \tau)}{(r-\mu)\beta_L} x_b^*
\end{aligned}
$$

C Proof of Proposition 8

We use the following notational convention: $c^*(\sigma)$ will denote the optimal coupon and we define $\xi(\sigma) = x_b^*/c = \frac{(\mu-r)\beta(\sigma)}{r(1-\beta(\sigma))}$.
Moreover, we write the incentive compatibility constraints as:

$$
\begin{aligned}
IC_L &= V_{L,L}^{(c,\delta,x_T)} - V_{L,H}^{(c,1,x_0)} \\
IC_H &= V_{H,L}^{(c,\delta,x_T)} - V_{H,H}^{(c,1,x_0)}
\end{aligned}
$$

A separating equilibrium requires that $IC_L > 0$ and $IC_H < 0$. We obtain for IC_L and IC_H the following representations:

$$
\begin{aligned}
IC_L &= \Lambda_1 - \Lambda_2^L - \Lambda_3(\sigma) + \Lambda_4 \\
IC_H &= \Lambda_1 - \Lambda_2^H - \Lambda_3(\sigma_H) + \Lambda_4,
\end{aligned}
$$

where

$$\Lambda_1 = \left(\frac{x_0}{c^*(\sigma_H)\xi(\sigma_H)}\right)^{\beta(\sigma_H)} \left(\frac{c^*(\sigma_H)}{r} - \frac{(1-\alpha)(1-\tau)c^*(\sigma_H)\xi(\sigma_H)}{r-\mu}\right) - \frac{\tau c^*(\sigma_H)}{r} \tag{24}$$

$$\Lambda_2^L = \left(\frac{x_0}{\min\{c^*(\sigma_H)\xi(\sigma), x_0\}}\right)^{\beta(\sigma)} \cdot \left(\frac{(1-\tau)c^*(\sigma_H)}{r} - \frac{(1-\tau)\min\{c^*(\sigma_H)\xi(\sigma), x_0\}}{r-\mu}\right) \tag{25}$$

$$\Lambda_2^H = \left(\frac{x_0}{c^*(\sigma_H)\xi(\sigma_H)}\right)^{\beta(\sigma_H)} \left(\frac{(1-\tau)c^*(\sigma_H)}{r} - \frac{(1-\tau)c^*(\sigma_H)\xi(\sigma_H)}{r-\mu}\right) \tag{26}$$

$$\Lambda_3(\cdot) = (1-\tau)\left(\frac{\hat{c}}{r}\left(\frac{x_0}{x_T}\right)^{\beta(\cdot)} - \left(\frac{x_0}{\hat{c}\xi(\cdot)}\right)^{\beta(\cdot)}\left(\frac{\hat{c}}{r} - \frac{\hat{c}\xi(\cdot)}{r-\mu}\right)\right) \tag{27}$$

$$\Lambda_4 = \left(\frac{\hat{c}}{r}\left(\frac{x_0}{x_T}\right)^{\beta(\sigma)} - \left(\frac{x_0}{\hat{c}\xi(\sigma)}\right)^{\beta(\sigma)}\left(\frac{\hat{c}}{r} - \frac{(1-\alpha)(1-\tau)\hat{c}\xi(\sigma)}{r-\mu}\right)\right) \tag{28}$$

IC_L and IC_H coincide in Λ_1 and Λ_4 and differ through Λ_2^L, Λ_2^H, and $\Lambda_3(\cdot)$. Note that the minimum function in Λ_2^L is due to the fact that for a large difference between σ and σ_H, $c^*(\sigma_H)\xi(\sigma)$ might be larger than x_0, which is not possible.[29] Without loss of generality, we can choose $\hat{c}$ as $c^*(\sigma)$. Therefore we need no similar restrictions in Λ_3 and Λ_4, since $c^*(\sigma)\xi(\sigma)$ and $c^*(\sigma)\xi(\sigma_H)$ take on well-defined limits.

Where appropriate, we indicate that the incentive compatibility constraints are functions of σ, σ_H, and x_T, i.e. $IC = IC(\sigma, \sigma_H, x_T)$.

Lemma 10 *(Characteristics)*

(i) For $x_T = x_0$,

$$IC_L(\sigma, \sigma, x_T) = IC_H(\sigma, \sigma, x_T) = 0.$$

(ii) For $x_T = x_0$,

$$\left.\frac{\partial IC_L(\sigma, \sigma_H, x_T)}{\partial \sigma_H}\right|_{\sigma_H=\sigma} = \left.\frac{\partial IC_H(\sigma, \sigma_H, x_T)}{\partial \sigma_H}\right|_{\sigma_H=\sigma} > 0.$$

(iii) For all $x_T \leq x_0$,

$$\left.\frac{\partial}{\partial \sigma_H}(IC_L(\sigma, \sigma_H, x_T) - IC_H(\sigma, \sigma_H, x_T))\right|_{\sigma_H=\sigma} \geq 0.$$

[29] In fact, for large σ_H and low σ, the equity claim for the type L firm is worthless.

(iv) For all $\sigma_H > \sigma$,

$$0 < \left.\frac{\partial IC_L(\sigma, \sigma_H, x_T)}{\partial x_T}\right|_{x_T = x_0} < \left.\frac{\partial IC_H(\sigma, \sigma_H, x_T)}{\partial x_T}\right|_{x_T = x_0}$$

For $\sigma_H = \sigma$, and for all x_T,

$$\frac{\partial IC_L(\sigma, \sigma, x_T)}{\partial x_T} = \frac{\partial IC_H(\sigma, \sigma, x_T)}{\partial x_T} > 0.$$

Proof

Part (i) follows directly from the definitions of IC_L and IC_H, since for $\sigma_H = \sigma$, Λ_2^L coincides with Λ_2^H, and $\Lambda_3(\sigma) = \Lambda_3(\sigma_H)$.

Part (ii): For $x_T = x_0$, set $\hat{c} = c^*(\sigma)$. Then tedious algebraic manipulations show that

$$\begin{aligned} \left.\frac{\partial IC_L}{\partial \sigma_H}\right|_{\sigma_H = \sigma} = \left.\frac{\partial IC_H}{\partial \sigma_H}\right|_{\sigma_H = \sigma} &= -\frac{x_0}{(r-\mu)\beta^2}\left(\frac{\tau}{\tau + z}\right)^{1-1/\beta}\frac{\partial \beta}{\partial \sigma} \cdot \\ &\quad \left[\log\left(\frac{\tau}{\tau+z}\right) + z\left(1 + \log\left(\frac{\tau}{\tau+z}\right)\right)\right], \end{aligned}$$

where $z = (\alpha(\tau - 1) - \tau)\beta > 0$. Since $\frac{\partial \beta}{\partial \sigma} > 0$, the term in the first line is negative. The term in squared brackets is negative for $z > 0$, so that $\left.\frac{\partial IC_L}{\partial \sigma_H}\right|_{\sigma_H = \sigma} = \left.\frac{\partial IC_H}{\partial \sigma_H}\right|_{\sigma_H = \sigma} > 0$ as asserted.

Part (iii): Note that $\frac{\partial}{\partial \sigma_H}(IC_L - IC_H) = \frac{\partial}{\partial \sigma_H}(\Lambda_2^H - \Lambda_2^L + \Lambda_3(\sigma_H))$. Tedious algebraic manipulations show that

$$\left.\frac{\partial}{\partial \sigma_H}\left(\Lambda_2^H - \Lambda_2^L + \Lambda_3(\sigma_H)\right)\right|_{\sigma_H = \sigma} = \frac{(1-\tau)c^*(\sigma)}{r}\left(\frac{x_0}{x_T}\right)^{\beta(\sigma)}\log\left(\frac{x_0}{x_T}\right)\frac{\partial \beta(\sigma)}{\partial \sigma}.$$

Since $\frac{\partial \beta(\sigma)}{\partial \sigma} > 0$, the above expression is always positive for $x_T < x_0$ and zero for $x_T = x_0$.

Part (iv): Taking the derivative of IC with respect to x_T yields

$$\left.\frac{\partial IC_L}{\partial x_T}\right|_{x_T=x_0} = \left.\frac{\partial(\Lambda_4 - \Lambda_3(\sigma))}{\partial x_T}\right|_{x_T=x_0} = -\frac{\beta(\sigma)}{x_0}\frac{\tau\hat{c}}{r} > 0$$

$$\left.\frac{\partial IC_H}{\partial x_T}\right|_{x_T=x_0} = \left.\frac{\partial(\Lambda_4 - \Lambda_3(\sigma_H))}{\partial x_T}\right|_{x_T=x_0}$$

$$= \frac{\hat{c}}{r}\left(\frac{(1-\tau)\beta(\sigma_H) - \beta(\sigma)}{x_0}\right) > 0,$$

where the last inequality follows from the fact that $\beta(\sigma) < \beta(\sigma_H) < 0$ holds, for all feasible combinations (σ, σ_H) with $\sigma_H > \sigma$. From this, it is easily verified that the difference is

$$\left.\frac{\partial IC_L}{\partial x_T}\right|_{x_T=x_0} - \left.\frac{\partial IC_H}{\partial x_T}\right|_{x_T=x_0} = \frac{(1-\tau)\hat{c}}{rx_0}\left(\beta(\sigma) - \beta(\sigma_H)\right) < 0.$$

The second assertion in part (v) follows trivially from the last inequality, where the difference is zero for $\sigma_H = \sigma$.

Proof of Proposition 8

Consider IC_L and IC_H as functions of σ_H. From lemma 10 parts (i) and (ii), it follows that $IC_L(\sigma_H)$ and $IC_H(\sigma_H)$ coincide for $\sigma_H = \sigma$ and $x_T = x_0$, i.e. both functions start in the same point, i.e. $IC_L = IC_H = 0$, and have the same (positive) first derivative in σ_H. Shifting x_T downwards by an infinitesimal ϵ to $x_0 - \epsilon$, shifts IC at $\sigma_H = \sigma$ in the negative, i.e. $IC_L = IC_H < 0$, and from part (iii) and (iv) it follows that there exists an infinitesimal interval $(\sigma, \sigma + \epsilon)$, where there does not exist a $\sigma_H \in (\sigma, \sigma + \epsilon)$ for which $IC_L \leq IC_H < 0$. Then there must exist a $\sigma_H \in (\sigma, \sigma + \epsilon)$ for which $IC_L > 0 > IC_H$, i.e. the incentive compatibility conditions are fulfilled and the assertion follows.

D Proof of Proposition 9

Assume that the triple $(c, \hat{c}, x_T)$ satisfies the incentive compatibility constraints, where $V_{H,H}^{(c,1,x_0)} \geq V_{H,L}^{(c,\delta,x_T)}$ holds with equality, i.e. $IC_H = 0$, where $IC_H = V_{H,H}^{(c,1,x_0)} - V_{H,L}^{(c,\delta,x_T)}$. The idea of the proof is to see what happens to the value $V_{L,L}^{(c,\delta,x_T)}$ of a type L firm, if we vary c and x_T such that still $IC_H = 0$ holds.
From the implicit function theorem, consider the *substitution rate* between c and

x_T so that the firm value remains unaffected. For type H and L, this approach results in the following representations

$$\frac{\mathrm{d}c(H)}{\mathrm{d}x_T(H)} = -\frac{\frac{\partial V_{H,L}}{\partial x_T}}{\frac{\partial V_{H,L}}{\partial c}}, \qquad \frac{\mathrm{d}c(L)}{\mathrm{d}x_T(L)} = -\frac{\frac{\partial V_{L,L}}{\partial x_T}}{\frac{\partial V_{L,L}}{\partial c}},$$

where $V_{H,L}$ and $V_{L,L}$ is shorthand notation for $V_{H,L}^{(c,\delta,x_T)}$ and $V_{L,L}^{(c,\delta,x_T)}$. Denote this as $SR_H = \frac{\mathrm{d}c(H)}{\mathrm{d}x_T(H)}$ and $SR_L = \frac{\mathrm{d}c(L)}{\mathrm{d}x_T(L)}$.

To establish the result, we need to show that $|SR_L| > |SR_H|$, or $\frac{|SR_H|}{|SR_L|} < 1$ for any choice of $\hat{c}$ holds. Since both SR_L and SR_H are negative, they indicate that the same firm value can be obtained with a higher trigger threshold x_T and an appropriately-chosen lower coupon c. If the relationship $|SR_L| > |SR_H|$ is satisfied, an increase of x_T and a decrease of c so that the firm value of the H type firm remains constant means that the firm value of an L type firm rises. This is due to the fact that for an L type firm a more pronounced decrease of c is required to keep the firm value constant. As a consequence of the fact that the firm value increases with c, the firm value of the L type firm can be increased and the incentive conditions remain satisfied. Therefore, an increase of x_T together with an appropriate decrease of c always adds firm value to the L type firm. These changes are feasible until either the coupon c attains zero or the step-up threshold x_T hits the current cash-flow level x_0. Therefore, in any optimal case, we can characterize the step-up bond (c, δ, x_T) as a bond with $c = 0$ that starts paying a positive coupon at x_T.

From the definitions of $V_{H,L}$ and $V_{L,L}$, we find the following derivatives:

$$\begin{aligned}
\frac{\partial V_{L,L}}{\partial c} &= \frac{\tau}{r}\left(1 - \left(\frac{x_0}{x_T}\right)^{\beta}\right), \\
\frac{\partial V_{H,L}}{\partial c} &= \frac{\partial V_{L,L}}{\partial c} + \frac{1-\tau}{r}\left(\left(\frac{x_0}{x_T}\right)^{\beta_H} - \left(\frac{x_0}{x_T}\right)^{\beta}\right), \\
\frac{\partial V_{L,L}}{\partial x_T} &= \frac{(c-\hat{c})\,\tau\beta_L\left(\frac{x_0}{x_T}\right)^{\beta_L}}{r\,x_T}, \\
\frac{\partial V_{H,L}}{\partial x_T} &= \frac{\partial V_{L,L}}{\partial x_T} + \frac{(c-\hat{c})\,(1-\tau)\left(\beta_L\left(\frac{x_0}{x_T}\right)^{\beta_L} - \beta_H\left(\frac{x_0}{x_T}\right)^{\beta_H}\right)}{r\,x_T}.
\end{aligned}$$

With these derivatives, $\frac{|SR_H|}{|SR_L|} < 1$ is equivalent to

$$\begin{aligned} \frac{|SR_H|}{|SR_L|} &= \frac{\frac{\partial V_{H,L}}{\partial x_T} \Big/ \frac{\partial V_{L,L}}{\partial x_T}}{\frac{\partial V_{H,L}}{\partial c} \Big/ \frac{\partial V_{L,L}}{\partial c}} < 1 \\ \Leftrightarrow & \quad \frac{1 + \frac{1-\tau}{\tau} \frac{\beta_L \left(\frac{x_0}{x_T}\right)^{\beta_L} - \beta_H \left(\frac{x_0}{x_T}\right)^{\beta_H}}{\beta_L \left(\frac{x_0}{x_T}\right)^{\beta_L}}}{1 + \frac{1-\tau}{\tau} \frac{\left(\frac{x_0}{x_T}\right)^{\beta_H} - \left(\frac{x_0}{x_T}\right)^{\beta_L}}{1 - \left(\frac{x_0}{x_T}\right)^{\beta_L}}} < 1 \end{aligned}$$

Note that $\frac{|SR_H|}{|SR_L|}$ is independent of c and $\hat{c}$. By straightforward algebraic manipulations the above inequality can be written as

$$\frac{1}{\beta_H} \left(1 - \left(\frac{x_0}{x_T}\right)^{-\beta_H}\right) < \frac{1}{\beta_L} \left(1 - \left(\frac{x_0}{x_T}\right)^{-\beta_L}\right). \qquad (29)$$

One can show that the derivative of $\frac{1}{\beta}\left(1 - \left(\frac{x_0}{x_T}\right)^{-\beta}\right)$ with respect to β

$$\frac{\partial}{\partial \beta} \left(\frac{1}{\beta} - \frac{1}{\beta} \left(\frac{x_0}{x_T}\right)^{-\beta}\right) = \frac{1}{\beta^2} \left(\frac{x_0}{x_T}\right)^{-\beta} \left[\left(1 - \left(\frac{x_0}{x_T}\right)^{\beta}\right) + \log\left(\left(\frac{x_0}{x_T}\right)^{\beta}\right)\right]$$

is always negative. This is due to the fact that the sign of the derivative comes from the term in squared brackets. Since the probability-weighted discount factor $\mathcal{D} = \left(\frac{x_0}{x_T}\right)^{\beta}$ always lies in the interval $\mathcal{D} \in (0,1)$ for $x_T < x_0$, the term $(1-\mathcal{D}) + \log \mathcal{D}$ is negative such as the derivative of the relevant term. Since $\beta_L < \beta_H$, inequality (29) is always fulfilled, and the assertion follows.

References

Anderson, R. W. and Sundaresan, S. (1996). Design and valuation of debt contracts. *The Review of Financial Studies*, 9(1):37–68.

Andrade, G. and Kaplan, S. N. (1998). How costly is financial (not economic) distress? evidence from highly leveraged transactions that became distressed. *Journal of Finance*, LIII(5):1443–1493.

Bhanot, K. and Mello, A. S. (2006). Should corporate debt include a rating trigger. *Journal of Financial Economics*, 79(1):69–98.

Bolton, P. and Dewatripont, M. (2005). *Contract Theory.* The MIT Press, Cambridge, Massachusetts.

Cho, I.-K. and Kreps, D. M. (1987). Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102(2):179–221.

Dixit, A. K. (1993). *The Art of Smooth Pasting.* Fundamentals of Pure and Applied Economics Publishers, Chur.

Dixit, A. K. and Pindyck, R. S. (1994). *Investment under Uncertainty.* Princeton University Press, Princeton, New Jersey.

Ericsson, J. (2000). Asset substitution, debt pricing, optimal leverage and maturity. *working paper*, pages McGill University, Montreal.

Fan, H. and Sundaresan, S. M. (2000). Debt valuation, renegotiation, and optimal dividend policy. *The Review of Financial Studies*, 13(4):1057–1099.

Fischer, E. O., Heinkel, R., and Zechner, J. (1989). Dynamic capital structure choice: Theory and tests. *Journal of Finance*, XLIV(1):19–40.

Flor, C. R. (2006). Asset substitution and debt renegotiation. *working paper.*

Goldstein, R., Ju, N., and Leland, H. E. (2001). An ebit-based model of dynamic capital structure. *Journal of Business*, 74(4):483–512.

Hackbarth, D., Hennessy, C. A., and Leland, H. E. (2007). Can the tradeoff theory explain debt structure? *The Review of Financial Studies*, 20(5):1389–1428.

Houweling, P., Mentink, A., and Vorst, T. (2004). Valuing euro rating-triggered step-up telecom bonds. *Journal of Derivatives*, 11(3):63–80.

Huang, J.-Z. and Huang, M. (2002). How much of the corporate-treasury yield spread is due to credit risk? *working paper*, page Stanford University.

Koziol, C. (2006). Why adding firm value with a put feature in debt contracts is better than renegotiation. *working paper*.

Koziol, C. (2007). Do good or bad borrowers pledge more collateral? *International Journal of Managerial Finance*, 3(2):132–163.

Lando, D. and Mortensen, A. (2004). On the pricing of step-up bonds in the european telecom sector. *Journal of Credit Risk*, 1(1).

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance*, XLIX(4):1213–1252.

Leland, H. E. (1998). Agency costs, risk management, and capital structure. *Journal of Finance*, 53(4):1213–1243.

Manso, G., Strulovici, B., and Tchistyi, A. (2007). Performance-sensitive debt. *working paper*, MIT.

Mella-Barral, P. (1999). The dynamics of default and debt renegotiation. *Review of Financial Studies*, 12(3):535–578.

Morellec, E. (2001). Asset liquidity, capital structure, and secured debt. *Journal of Financial Economics*, 61(2):173–206.

Morellec, E. (2004). Can managerial discretion explain observed leverage ratios? *Review of Financial Studies*, 17(1):257–294.

Nachman, D. C. and Noe, T. H. (1994). Optimal design of securities under asymmetric information. *The Review of Financial Studies*, 7(1):1–44.

Ross, S. A. (1977). The determination of financial structure: The incentive-signalling approach. *Bell Journal of Economics*, 8(1):23–40.

Silva, S. and Pereira, J. A. (2007). Optimal debt, asset substitution and coupon rating-trigger covenants. *working paper*, page Portucalense University.

Strebulaev, I. A. (2007). Do tests of capital structure theory mean what they say? *Journal of Finance*, 62(4):1747–1787.